Expressions and Equations

You can use algebraic expressions and equations to model and analyze real-world situations. In this unit, you will learn about expressions, equations, and graphs.

Chapter 1
The Language of Algebra

Chapter 2
Real Numbers

Chapter 3
Solving Linear Equations
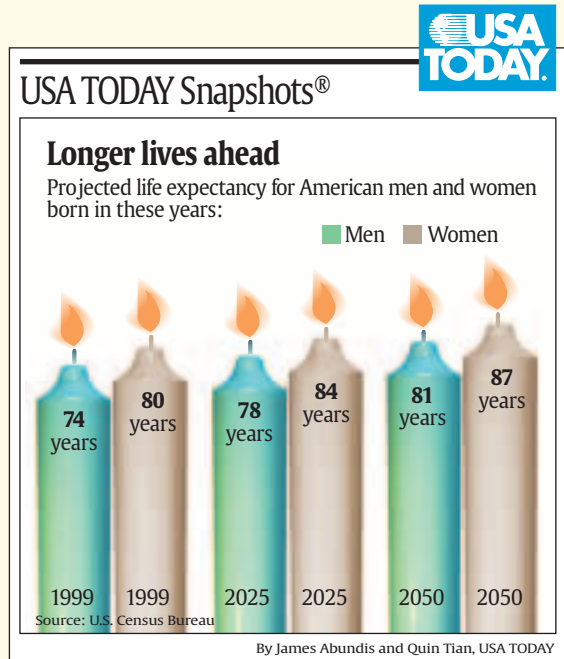
CONTENTS

WebQuest Internet Project

Can You Fit 100 Candles on a Cake?

"The mystique of living to be 100 will be lost by the year 2020 as 100th birthdays become commonplace, predicts Mike Parker, assistant professor of social work, University of Alabama, Tuscaloosa, and a gerontologist specializing in successful aging. He says that, in the 21st century, the fastest growing age group in the country will be centenarians—those who live 100 years or longer." In this project, you will explore how equations, functions, and graphs can help represent aging and population growth.

Log on to **www.algebra1.com/webquest**.
Begin your WebQuest by reading the Task.

Then continue working on your WebQuest as you study Unit 1.

Lesson	1-9	2-6	3-6
Page	55	100	159

USA TODAY Snapshots®

Longer lives ahead

Projected life expectancy for American men and women born in these years:

Men   Women

| 74 years | 80 years | 78 years | 84 years | 81 years | 87 years |
| 1999 | 1999 | 2025 | 2025 | 2050 | 2050 |

Source: U.S. Census Bureau

By James Abundis and Quin Tian, USA TODAY

CONTENTS

The Language of Algebra

What You'll Learn

- **Lesson 1-1** Write algebraic expressions.
- **Lessons 1-2 and 1-3** Evaluate expressions and solve open sentences.
- **Lessons 1-4 through 1-6** Use algebraic properties of identity and equality.
- **Lesson 1-7** Use conditional statements and counterexamples.
- **Lessons 1-8 and 1-9** Interpret graphs of functions and analyze data in statistical graphs.

Key Vocabulary

- **variable** (p. 6)
- **order of operations** (p. 11)
- **identity** (p. 21)
- **like terms** (p. 28)
- **counterexample** (p. 38)

Why It's Important

In every state and in every country, you find unique and inspiring architecture. Architects can use algebraic expressions to describe the volume of the structures they design. A few of the shapes these buildings can resemble are a rectangle, a pentagon, or even a pyramid. *You will find the amount of space occupied by a pyramid in Lesson 1-2.*

CONTENTS

Getting Started

▶ **Prerequisite Skills** To be successful in this chapter, you'll need to master these skills and be able to apply them in problem-solving situations. Review these skills before beginning Chapter 1.

For Lessons 1-1, 1-2, and 1-3   Multiply and Divide Whole Numbers

Find each product or quotient.

1. $8 \cdot 8$ **2.** $4 \cdot 16$ **3.** $18 \cdot 9$ **4.** $23 \cdot 6$

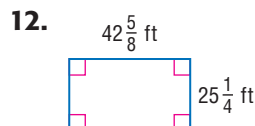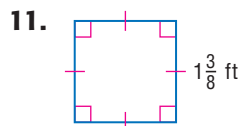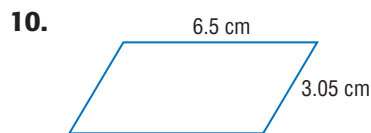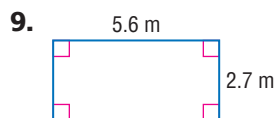**5.** $57 \div 3$ **6.** $68 \div 4$ **7.** $\dfrac{72}{3}$ **8.** $\dfrac{90}{6}$

For Lessons 1-1, 1-2, 1-5, and 1-6   Find Perimeter

Find the perimeter of each figure. *(For review, see pages 813 and 814.)*

9.

5.6 m

2.7 m

10.

6.5 cm

3.05 cm

11.

$1\frac{3}{8}$ ft

12.

$42\frac{5}{8}$ ft

$25\frac{1}{4}$ ft

For Lessons 1-5 and 1-6   Multiply and Divide Decimals and Fractions

Find each product or quotient. *(For review, see pages 800 and 801.)*

13. $6 \cdot 1.2$ **14.** $0.5 \cdot 3.9$ **15.** $3.24 \div 1.8$ **16.** $10.64 \div 1.4$

17. $\dfrac{3}{4} \cdot 12$ **18.** $1\dfrac{2}{3} \cdot \dfrac{3}{4}$ **19.** $\dfrac{5}{16} \div \dfrac{9}{12}$ **20.** $\dfrac{5}{6} \div \dfrac{2}{3}$
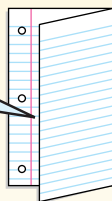
FOLDABLES™

Study Organizer

Make this Foldable to help you organize information about algebraic properties. Begin with a sheet of notebook paper.
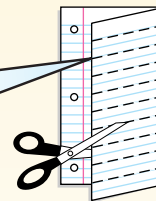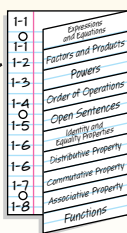
Step 1 Fold

Fold lengthwise to the holes.

Step 2 Cut

Cut along the top line and then cut 9 tabs.

Step 3 Label

Label the tabs using the lesson numbers and concepts.

1-1 Expressions and Equations
1-1 Factors and Products
1-2 Powers
1-3 Order of Operations
1-4 Open Sentences
1-5 Identity and Equality Properties
1-6 Distributive Property
1-6 Commutative Property
1-7 Associative Property
1-8 Functions

Reading and Writing Store the Foldable in a 3-ring binder. As you read and study the chapter, write notes and examples under the tabs.
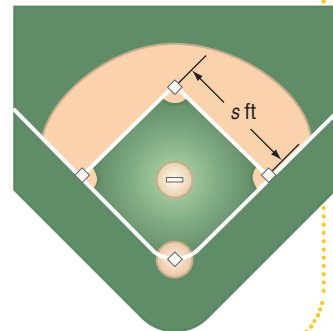
1-1 Variables and Expressions

Vocabulary
- variables
- algebraic expression
- factors
- product
- power
- base
- exponent
- evaluate

What You'll Learn

- Write mathematical expressions for verbal expressions.
- Write verbal expressions for mathematical expressions.

What expression can be used to find the perimeter of a baseball diamond?

A baseball infield is a square with a base at each corner. Each base lies the same distance from the next one. Suppose s represents the length of each side of the square. Since the infield is a square, you can use the expression 4 times s, or $4s$ to find the perimeter of the square.

s ft

WRITE MATHEMATICAL EXPRESSIONS In the algebraic expression $4s$, the letter s is called a variable. In algebra, **variables** are symbols used to represent unspecified numbers or values. Any letter may be used as a variable. *The letter s was used above because it is the first letter of the word side.*

An **algebraic expression** consists of one or more numbers and variables along with one or more arithmetic operations. Here are some examples of algebraic expressions.

$$5x \qquad 3x - 7 \qquad 4 + \frac{p}{q} \qquad m \times 5n \qquad 3ab \div 5cd$$

In algebraic expressions, a raised dot or parentheses are often used to indicate multiplication as the symbol $\times$ can be easily mistaken for the letter x. Here are several ways to represent the product of x and y.

$$xy \qquad x \cdot y \qquad x(y) \qquad (x)y \qquad (x)(y)$$

In each expression, the quantities being multiplied are called **factors**, and the result is called the **product**.

It is often necessary to translate verbal expressions into algebraic expressions.

Example 1 Write Algebraic Expressions

Write an algebraic expression for each verbal expression.

a. eight more than a number n

The words *more than* suggest addition.

$$\underbrace{\text{eight}}_{8} \quad \underbrace{\text{more than}}_{+} \quad \underbrace{\text{a number } n}_{n}$$

Thus, the algebraic expression is $8 + n$.

CONTENTS

b. the difference of 7 and 4 times a number x

Difference implies subtract, and *times* implies multiply. So the expression can be written as $7 - 4x$.

c. one third of the size of the original area a

The word *of* implies multiply, so the expression can be written as $\frac{1}{3}a$ or $\frac{a}{3}$.

An expression like x^n is called a **power** and is read "x to the nth power." The variable x is called the **base**, and n is called the **exponent**. The exponent indicates the number of times the base is used as a factor.

Symbols	Words	Meaning
3^1	3 to the first power	3
3^2	3 to the second power or 3 squared	$3 \cdot 3$
3^3	3 to the third power or 3 cubed	$3 \cdot 3 \cdot 3$
3^4	3 to the fourth power	$3 \cdot 3 \cdot 3 \cdot 3$
$2b^6$	2 times b to the sixth power	$2 \cdot b \cdot b \cdot b \cdot b \cdot b \cdot b$
x^n	x to the nth power	$\underbrace{x \cdot x \cdot x \cdot \ldots \cdot x}_{n \text{ factors}}$

By definition, for any nonzero number x, $x^0 = 1$.

Example 2 Write Algebraic Expressions with Powers

Write each expression algebraically.

a. the product of 7 and m to the fifth power

$7m^5$

b. the difference of 4 and x squared

$4 - x^2$

To **evaluate** an expression means to find its value.

Example 3 Evaluate Powers

Evaluate each expression.

a. 2^6

$2^6 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$ Use 2 as a factor 6 times.

$\quad = 64$ Multiply.

b. 4^3

$4^3 = 4 \cdot 4 \cdot 4$ Use 4 as a factor 3 times.

$\quad = 64$ Multiply.

WRITE VERBAL EXPRESSIONS   Another important skill is translating algebraic expressions into verbal expressions.

Example 4 Write Verbal Expressions

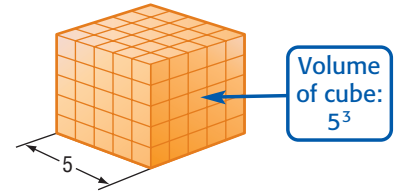Write a verbal expression for each algebraic expression.

a. $4m^3$

the product of 4 and m to the third power

**b. $c^2 + 21d$**

the sum of c squared and 21 times d

www.algebra1.com/extra_examples/nc

CONTENTS

c. 5^3

five to the third power or five cubed

Volume of cube: 5^3

Check for Understanding

Concept Check
1. **Explain** the difference between an algebraic expression and a verbal expression.
2. **Write** an expression that represents the perimeter of the rectangle.
3. **OPEN ENDED** Give an example of a variable to the fifth power.
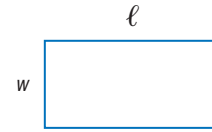
Guided Practice
Write an algebraic expression for each verbal expression.
4. the sum of j and 13
5. 24 less than three times a number

Evaluate each expression.
6. 9^2
7. 4^4

Write a verbal expression for each algebraic expression.
8. $4m^4$
9. $\frac{1}{2}n^3$

Application
10. **MONEY** Lorenzo bought several pounds of chocolate-covered peanuts and gave the cashier a $20 bill. Write an expression for the amount of change he will receive if p represents the cost of the peanuts.

Practice and Apply

Homework Help

| For Exercises	See Examples
11–18	1, 2
21–28	3
31–42	4

Extra Practice
See page 820.

Write an algebraic expression for each verbal expression.
11. the sum of 35 and z
12. the sum of a number and 7
13. the product of 16 and p
14. the product of 5 and a number
15. 49 increased by twice a number
16. 18 and three times d
17. two-thirds the square of a number
18. one-half the cube of n

19. **SAVINGS** Kendra is saving to buy a new computer. Write an expression to represent the amount of money she will have if she has s dollars saved and she adds d dollars per week for the next 12 weeks.

20. **GEOMETRY** The area of a circle can be found by multiplying the number π by the square of the radius. If the radius of a circle is r, write an expression that represents the area of the circle.
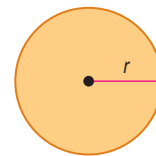
Evaluate each expression.
21. 6^2
22. 8^2
23. 3^4
24. 6^3
25. 3^5
26. 15^3
27. 10^6
28. 100^3

29. **FOOD** A bakery sells a dozen bagels for $8.50 and a dozen donuts for $3.99. Write an expression for the cost of buying b dozen bagels and d dozen donuts.

CONTENTS

30. TRAVEL Before starting her vacation, Sari's car had 23,500 miles on the odometer. She drives an average of m miles each day for two weeks. Write an expression that represents the mileage on Sari's odometer after her trip.

Write a verbal expression for each algebraic expression.

31. $7p$

32. $15r$

33. 3^3

34. 5^4

35. $3x^2 + 4$

36. $2n^3 + 12$

37. $a^4 \cdot b^2$

38. $n^3 \cdot p^5$

39. $\dfrac{12z^2}{5}$

40. $\dfrac{8g^3}{4}$

41. $3x^2 - 2x$

42. $4f^5 - 9k^3$

43. PHYSICAL SCIENCE When water freezes, its volume is increased by one-eleventh. In other words, the volume of ice equals the sum of the volume of the water and the product of one-eleventh and the volume of the water. If x cubic centimeters of water is frozen, write an expression for the volume of the ice that is formed.

44. GEOMETRY The surface area of a rectangular prism is the sum of:
- the product of twice the length ℓ and the width w,
- the product of twice the length and the height h, and
- the product of twice the width and the height.
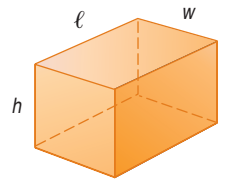
Write an expression that represents the surface area of a prism.

45. RECYCLING Each person in the United States produces approximately 3.5 pounds of trash each day. Write an expression representing the pounds of trash produced in a day by a family that has m members. **Source:** *Vitality*

46. CRITICAL THINKING In the square, the variable a represents a positive whole number. Find the value of a such that the area and the perimeter of the square are the same.

47. WRITING IN MATH Answer the question that was posed at the beginning of the lesson.

What expression can be used to find the perimeter of a baseball diamond?
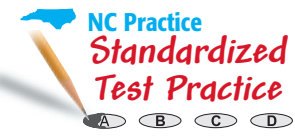
Include the following in your answer:
- two different verbal expressions that you can use to describe the perimeter of a square, and
- an algebraic expression other than $4s$ that you can use to represent the perimeter of a square.

48. What is 6 more than 2 times a certain number x?

Ⓐ $2x - 6$　　　Ⓑ $2x$　　　Ⓒ $6x - 2$　　　Ⓓ $2x + 6$

49. Write $4 \cdot 4 \cdot 4 \cdot c \cdot c \cdot c \cdot c$ using exponents.

Ⓐ $3^4 4^c$　　　Ⓑ $4^3 c^4$　　　Ⓒ $(4c)^7$　　　Ⓓ $4c$

Maintain Your Skills

Getting Ready for the Next Lesson

PREREQUISITE SKILL **Evaluate each expression.**
*(To review **operations with fractions**, see pages 798–801.)*

50. $14.3 + 1.8$

51. $10 - 3.24$

52. 1.04×4.3

53. $15.36 \div 4.8$

54. $\dfrac{1}{3} + \dfrac{2}{5}$

55. $\dfrac{3}{4} - \dfrac{1}{6}$

56. $\dfrac{3}{8} \times \dfrac{4}{9}$

57. $\dfrac{7}{10} \div \dfrac{3}{5}$

Reading Mathematics

Translating from English to Algebra

You learned in Lesson 1-1 that it is often necessary to translate words into algebraic expressions. Generally, there are "clue" words such as *more than*, *times*, *less than*, and so on, which indicate the operation to use. These words also help to connect numerical data. The table shows a few examples.

Words	Algebraic Expression
four times x plus y	$4x + y$
four times the sum of x and y	$4(x + y)$
four times the quantity x plus y	$4(x + y)$

Notice that all three expressions are worded differently, but the first expression is the only one that is different algebraically. In the second expression, parentheses indicate that the *sum*, $x + y$, is multiplied by four. In algebraic expressions, terms grouped by parentheses are treated as one quantity. So, $4(x + y)$ can also be read as *four times the quantity x plus y.*

Words that may indicate parentheses are *sum*, *difference*, *product*, and *quantity*.

Reading to Learn

Read each verbal expression aloud. Then match it with the correct algebraic expression.

1. nine divided by 2 plus n

2. four divided by the difference of n and six

3. n plus five squared

4. three times the quantity eight plus n

5. nine divided by the quantity 2 plus n

6. three times eight plus n

7. the quantity n plus five squared

8. four divided by n minus six

a. $(n + 5)^2$
b. $4 \div (n - 6)$
c. $9 \div 2 + n$
d. $3(8) + n$
e. $4 \div n - 6$
f. $n + 5^2$
g. $9 \div (2 + n)$
h. $3(8 + n)$

Write each algebraic expression in words.

9. $5x + 1$

10. $5(x + 1)$

11. $3 + 7x$

12. $(3 + x) \cdot 7$

13. $(6 + b) \div y$

14. $6 + (b \div y)$

Order of Operations

Vocabulary
- order of operations

What You'll Learn

- Evaluate numerical expressions by using the order of operations.
- Evaluate algebraic expressions by using the order of operations.

How is the monthly cost of internet service determined?

Nicole is signing up with a new internet service provider. The service costs $4.95 a month, which includes 100 hours of access. If she is online for more than 100 hours, she must pay an additional $0.99 per hour. Suppose Nicole is online for 117 hours the first month. The expression $4.95 + 0.99(117 - 100)$ represents what Nicole must pay for the month.

@home.net

$4.95 per month*
- includes 100 free hours
- accessible anywhere**

*0.99 per hour after 100 hours
**Requires v.95 net modem

EVALUATE RATIONAL EXPRESSIONS Numerical expressions often contain more than one operation. A rule is needed to let you know which operation to perform first. This rule is called the **order of operations**.

Key Concept — *Order of Operations*

Step 1 Evaluate expressions inside grouping symbols.
Step 2 Evaluate all powers.
Step 3 Do all multiplications and/or divisions from left to right.
Step 4 Do all additions and/or subtractions from left to right.

Example 1 Evaluate Expressions

Evaluate each expression.

a. $3 + 2 \cdot 3 + 5$

$$3 + 2 \cdot 3 + 5 = 3 + 6 + 5 \quad \text{Multiply 2 and 3.}$$
$$= 9 + 5 \quad \text{Add 3 and 6.}$$
$$= 14 \quad \text{Add 9 and 5.}$$

b. $15 \div 3 \cdot 5 - 4^2$

$$15 \div 3 \cdot 5 - 4^2 = 15 \div 3 \cdot 5 - 16 \quad \text{Evaluate powers.}$$
$$= 5 \cdot 5 - 16 \quad \text{Divide 15 by 3.}$$
$$= 25 - 16 \quad \text{Multiply 5 by 5.}$$
$$= 9 \quad \text{Subtract 16 from 25.}$$

CONTENTS

Grouping symbols such as parentheses ( ), brackets [ ], and braces { } are used to clarify or change the order of operations. They indicate that the expression within the grouping symbol is to be evaluated first.

Example 2 Grouping Symbols

Evaluate each expression.

a. 2(5) + 3(4 + 3)

$$2(5) + 3(4 + 3) = 2(5) + 3(7)$$ Evaluate inside grouping symbols.

$$= 10 + 21$$ Multiply expressions left to right.

$$= 31$$ Add 10 and 21.

b. $2[5 + (30 \div 6)^2]$

$$2[5 + (30 \div 6)^2] = 2[5 + (5)^2]$$ Evaluate innermost expression first.

$$= 2[5 + 25]$$ Evaluate power inside grouping symbol.

$$= 2[30]$$ Evaluate expression in grouping symbol.

$$= 60$$ Multiply.

A fraction bar is another type of grouping symbol. It indicates that the numerator and denominator should each be treated as a single value.

Example 3 Fraction Bar

Evaluate $\dfrac{6 + 4^2}{3^2 \cdot 4}$.

$\dfrac{6 + 4^2}{3^2 \cdot 4}$ means $(6 + 4^2) \div (3^2 \cdot 4)$.

$$\frac{6 + 4^2}{3^2 \cdot 4} = \frac{6 + 16}{3^2 \cdot 4}$$ Evaluate the power in the numerator.

$$= \frac{22}{3^2 \cdot 4}$$ Add 6 and 16 in the numerator.

$$= \frac{22}{9 \cdot 4}$$ Evaluate the power in the denominator.

$$= \frac{22}{36} \text{ or } \frac{11}{18}$$ Multiply 9 and 4 in the denominator. Then simplify.

EVALUATE ALGEBRAIC EXPRESSIONS   Like numerical expressions, algebraic expressions often contain more than one operation. Algebraic expressions can be evaluated when the values of the variables are known. First, replace the variables with their values. Then, find the value of the numerical expression using the order of operations.

Example 4 Evaluate an Algebraic Expression

Evaluate $a^2 - (b^3 - 4c)$ if $a = 7$, $b = 3$, and $c = 5$.

$$a^2 - (b^3 - 4c) = 7^2 - (3^3 - 4 \cdot 5)$$ Replace a with 7, b with 3, and c with 5.

$$= 7^2 - (27 - 4 \cdot 5)$$ Evaluate 3^3.

$$= 7^2 - (27 - 20)$$ Multiply 4 and 5.

$$= 7^2 - 7$$ Subtract 20 from 27.

$$= 49 - 7$$ Evaluate 7^2.

$$= 42$$ Subtract.

CONTENTS

Example 5 *Use Algebraic Expressions*

ARCHITECTURE The Pyramid Arena in Memphis, Tennessee, is the third largest pyramid in the world. The area of its base is 360,000 square feet, and it is 321 feet high. The volume of any pyramid is one third of the product of the area of the base B and its height h.

a. Write an expression that represents the volume of a pyramid.

one third	of	the product of area of base and height	
$\frac{1}{3}$	$\times$	$(B \cdot h)$	or $\frac{1}{3}Bh$

b. Find the volume of the Pyramid Arena.

Evaluate $\frac{1}{3}(Bh)$ for $B = 360{,}000$ and $h = 321$.

$\frac{1}{3}(Bh) = \frac{1}{3}(360{,}000 \cdot 321)$ *B = 360,000 and h = 321*

$= \frac{1}{3}(115{,}560{,}000)$ *Multiply 360,000 by 321.*

$= \frac{115{,}560{,}000}{3}$ *Multiply $\frac{1}{3}$ by 115,560,000.*

$= 38{,}520{,}000$ *Divide 115,560,000 by 3.*

The volume of the Pyramid Arena is 38,520,000 cubic feet.

Check for Understanding

Concept Check
1. **Describe** how to evaluate $8[6^2 - 3(2 + 5)] \div 8 + 3$.

2. **OPEN ENDED** Write an expression involving division in which the first step in evaluating the expression is addition.

3. **FIND THE ERROR** Laurie and Chase are evaluating $3[4 + (27 \div 3)]^2$.

Laurie
$3[4 + (27 \div 3)]^2 = 3(4 + 9^2)$
$= 3(4 + 81)$
$= 3(85)$
$= 255$

Chase
$3[4 + (27 \div 3)]^2 = 3(4 + 9)^2$
$= 3(13)^2$
$= 3(169)$
$= 507$

Who is correct? Explain your reasoning.

Guided Practice
Evaluate each expression.

4. $(4 + 6)7$
5. $50 - (15 + 9)$
6. $29 - 3(9 - 4)$
7. $[7(2) - 4] + [9 + 8(4)]$
8. $\dfrac{(4 \cdot 3)^2 \cdot 5}{9 + 3}$
9. $\dfrac{3 + 2^3}{5^2(4)}$

Evaluate each expression if $g = 4$, $h = 6$, $j = 8$, and $k = 12$.

10. $hk - gj$
11. $2k + gh^2 - j$
12. $\dfrac{2g(h - g)}{gh - j}$

Application
SHOPPING For Exercises 13 and 14, use the following information.
A computer store has certain software on sale at 3 for $20.00, with a limit of 3 at the sale price. Additional software is available at the regular price of $9.95 each.

13. Write an expression you could use to find the cost of 5 software packages.

14. How much would 5 software packages cost?

Practice and Apply

Evaluate each expression.

15. $(12 - 6) \cdot 2$

16. $(16 - 3) \cdot 4$

17. $15 + 3 \cdot 2$

18. $22 + 3 \cdot 7$

19. $4(11 + 7) - 9 \cdot 8$

20. $12(9 + 5) - 6 \cdot 3$

21. $12 \div 3 \cdot 5 - 4^2$

22. $15 \div 3 \cdot 5 - 4^2$

23. $288 \div [3(9 + 3)]$

Evaluate each expression.
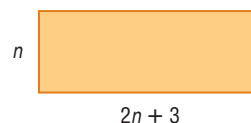
24. $390 \div [5(7 + 6)]$

25. $\dfrac{2 \cdot 8^2 - 2^2 \cdot 8}{2 \cdot 8}$

26. $\dfrac{4 \cdot 6^2 - 4^2 \cdot 6}{4 \cdot 6}$

27. $\dfrac{[(8 + 5)(6 - 2)^2] - (4 \cdot 17 \div 2)}{[(24 \div 2) \div 3]}$

28. $6 - \left[\dfrac{2 + 7}{3} - (2 \cdot 3 - 5) \right]$

29. GEOMETRY Find the area of the rectangle when $n = 4$ centimeters.

n

$2n + 3$

ENTERTAINMENT **For Exercises 30 and 31, use the following information.**
Derrick and Samantha are selling tickets for their school musical. Floor seats cost $7.50 and balcony seats cost $5.00. Samantha sells 60 floor seats and 70 balcony seats, Derrick sells 50 floor seats and 90 balcony seats.

30. Write an expression to show how much money Samantha and Derrick have collected for tickets.

31. Evaluate the expression to determine how much they collected.

Evaluate each expression if $x = 12$, $y = 8$, and $z = 3$.

32. $x + y^2 + z^2$

33. $x^3 + y + z^3$

34. $3xy - z$

35. $4x - yz$

36. $\dfrac{2xy - z^3}{z}$

37. $\dfrac{xy^2 - 3z}{3}$

38. $\left(\dfrac{x}{y} \right)^2 - \dfrac{3y - z}{(x - y)^2}$

39. $\dfrac{x - z^2}{y \div x} + \dfrac{2y - x}{y^2 \div 2}$

40. BIOLOGY Most bacteria reproduce by dividing into identical cells. This process is called *binary fission*. A certain type of bacteria can double its numbers every 20 minutes. Suppose 100 of these cells are in one culture dish and 250 of the cells are in another culture dish. Write and evaluate an expression that shows the total number of bacteria cells in both dishes after 20 minutes.

BUSINESS **For Exercises 41–43, use the following information.**
Mr. Martinez is a sales representative for an agricultural supply company. He receives a salary and monthly commission. He also receives a bonus each time he reaches a sales goal.

41. Write a verbal expression that describes how much Mr. Martinez earns in a year if he receives four equal bonuses.

42. Let e represent earnings, s represent his salary, c represent his commission, and b represent his bonus. Write an algebraic expression to represent his earnings if he receives four equal bonuses.

43. Suppose Mr. Martinez's annual salary is $42,000 and his average commission is $825 each month. If he receives four bonuses of $750 each, how much does he earn in a year?

CONTENTS

44. CRITICAL THINKING Choose three numbers from 1 to 6. Write as many expressions as possible that have different results when they are evaluated. You must use all three numbers in each expression, and each can only be used once.

45. WRITING IN MATH Answer the question that was posed at the beginning of the lesson.

How is the monthly cost of internet service determined?

Include the following in your answer:
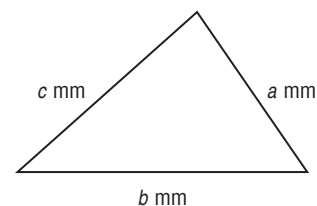- an expression for the cost of service if Nicole has a coupon for $25 off her base rate for her first six months, and
- an explanation of the advantage of using an algebraic expression over making a table of possible monthly charges.
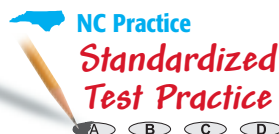
46. Find the perimeter of the triangle using the formula $P = a + b + c$ if $a = 10$, $b = 12$, and $c = 17$.

- Ⓐ 39 mm
- Ⓑ 19.5 mm
- Ⓒ 60 mm
- Ⓓ 78 mm

47. Evaluate $(5 - 1)^3 + (11 - 2)^2 + (7 - 4)^3$.

- Ⓐ 586
- Ⓑ 172
- Ⓒ 106
- Ⓓ 39

Graphing Calculator

EVALUATING EXPRESSIONS Use a calculator to evaluate each expression.

48. $\dfrac{0.25x^2}{7x^3}$ if $x = 0.75$

49. $\dfrac{2x^2}{x^2 - x}$ if $x = 27.89$

50. $\dfrac{x^3 + x^2}{x^3 - x^2}$ if $x = 12.75$

Maintain Your Skills

Mixed Review Write an algebraic expression for each verbal expression. *(Lesson 1-1)*

51. the product of the third power of a and the fourth power of b

52. six less than three times the square of y

53. the sum of a and b increased by the quotient of b and a

54. four times the sum of r and s increased by twice the difference of r and s

55. triple the difference of 55 and the cube of w

Evaluate each expression. *(Lesson 1-1)*

56. 2^4

57. 12^1

58. 8^2

59. 4^4

Write a verbal expression for each algebraic expression. *(Lesson 1-1)*

60. $5n + \dfrac{n}{2}$

61. $q^2 - 12$

62. $\dfrac{(x + 3)}{(x - 2)^2}$

63. $\dfrac{x^3}{9}$

Getting Ready for the Next Lesson

PREREQUISITE SKILL **Find the value of each expression.**
*(To review **operations with decimals and fractions**, see pages 798–801.)*

64. $0.5 - 0.0075$

65. $5.6 + 1.612$

66. $14.9968 \div 5.2$

67. $2.3(6.425)$

68. $4\dfrac{1}{8} - 1\dfrac{1}{2}$

69. $\dfrac{3}{5} + 2\dfrac{5}{7}$

70. $\dfrac{5}{6} \cdot \dfrac{4}{5}$

71. $8 \div \dfrac{2}{9}$

CONTENTS

Vocabulary
- open sentence
- solving an open sentence
- solution
- equation
- replacement set
- set
- element
- solution set
- inequality

What You'll Learn

- Solve open sentence equations.
- Solve open sentence inequalities.

How can you use open sentences to stay within a budget?

The Daily News sells garage sale kits. The Spring Creek Homeowners Association is planning a community garage sale, and their budget for advertising is $135. The expression $15.50 + 5n$ can be used to represent the cost of purchasing $n + 1$ kits. The open sentence $15.50 + 5n \leq 135$ can be used to ensure that the budget is met.

Garage sale kit includes:
- Weekend ad
- Signs
- Announcements
- Balloons
- Price stickers
- Sales sheet

COMPLETE PACKAGE $15.50

Additional kits available for $5.00 each

SOLVE EQUATIONS   A mathematical statement with one or more variables is called an **open sentence**. An open sentence is neither true nor false until the variables have been replaced by specific values. The process of finding a value for a variable that results in a true sentence is called **solving the open sentence**. This replacement value is called a **solution** of the open sentence. A sentence that contains an equals sign, $=$, is called an **equation**.

A set of numbers from which replacements for a variable may be chosen is called a **replacement set**. A **set** is a collection of objects or numbers. It is often shown using braces, { }, and is usually named by a capital letter. Each object or number in the set is called an **element**, or member. The **solution set** of an open sentence is the set of elements from the replacement set that make an open sentence true.

Example 1 Use a Replacement Set to Solve an Equation

Find the solution set for each equation if the replacement set is {3, 4, 5, 6, 7}.

a. $6n + 7 = 37$

Replace n in $6n + 7 = 37$ with each value in the replacement set.

n	$6n + 7 = 37$	True or False?
3	$6(3) + 7 \stackrel{?}{=} 37 \rightarrow 25 \neq 37$	false
4	$6(4) + 7 \stackrel{?}{=} 37 \rightarrow 31 \neq 37$	false
5	$6(5) + 7 \stackrel{?}{=} 37 \rightarrow 37 = 37$	true ✓
6	$6(6) + 7 \stackrel{?}{=} 37 \rightarrow 43 \neq 37$	false
7	$6(7) + 7 \stackrel{?}{=} 37 \rightarrow 49 \neq 37$	false

Since $n = 5$ makes the equation true, the solution of $6n + 7 = 37$ is 5.

The solution set is {5}.

CONTENTS

b. $5(x + 2) = 40$

Replace x in $5(x + 2) = 40$ with each value in the replacement set.

x	$5(x + 2) = 40$	True or False?
3	$5(3 + 2) \stackrel{?}{=} 40 \rightarrow 25 \neq 40$	false
4	$5(4 + 2) \stackrel{?}{=} 40 \rightarrow 30 \neq 40$	false
5	$5(5 + 2) \stackrel{?}{=} 40 \rightarrow 35 \neq 40$	false
6	$5(6 + 2) \stackrel{?}{=} 40 \rightarrow 40 = 40$	true ✓
7	$5(7 + 2) \stackrel{?}{=} 40 \rightarrow 45 \neq 40$	false

The solution of $5(x + 2) = 40$ is 6. The solution set is {6}.

You can often solve an equation by applying the order of operations.

Example 2 Use Order of Operations to Solve an Equation

Solve $\dfrac{13 + 2(4)}{3(5 - 4)} = q$.

$\dfrac{13 + 2(4)}{3(5 - 4)} = q$ Original equation

$\dfrac{13 + 8}{3(1)} = q$ Multiply 2 and 4 in the numerator.
Subtract 4 from 5 in the denominator.

$\dfrac{21}{3} = q$ Simplify.

$7 = q$ Divide.   The solution is 7.

SOLVE INEQUALITIES   An open sentence that contains the symbol $<$, $\leq$, $>$, or $\geq$ is called an **inequality**. Inequalities can be solved in the same way as equations.

Example 3 Find the Solution Set of an Inequality

Find the solution set for $18 - y < 10$ if the replacement set is {7, 8, 9, 10, 11, 12}.

Replace y in $18 - y < 10$ with each value in the replacement set.

y	$18 - y < 10$	True or False?
7	$18 - 7 \stackrel{?}{<} 10 \rightarrow 11 \not< 10$	false
8	$18 - 8 \stackrel{?}{<} 10 \rightarrow 10 \not< 10$	false
9	$18 - 9 \stackrel{?}{<} 10 \rightarrow 9 < 10$	true ✓
10	$18 - 10 \stackrel{?}{<} 10 \rightarrow 8 < 10$	true ✓
11	$18 - 11 \stackrel{?}{<} 10 \rightarrow 7 < 10$	true ✓
12	$18 - 12 \stackrel{?}{<} 10 \rightarrow 6 < 10$	true ✓

The solution set for $18 - y < 10$ is {9, 10, 11, 12}.

Example 4 Solve an Inequality

FUND-RAISING   Refer to the application at the beginning of the lesson. How many garage sale kits can the association buy and stay within their budget?

Explore   The association can spend no more than $135. So the situation can be represented by the inequality $15.50 + 5n \leq 135$.

(continued on the next page)

www.algebra1.com/extra_examples/nc

CONTENTS

Plan   Since no replacement set is given, estimate to find reasonable values for the replacement set.

Solve   Start by letting $n = 10$ and then adjust values up or down as needed.

$$15.50 + 5n \leq 135 \quad \text{Original inequality}$$

$$15.50 + 5(10) \leq 135 \quad n = 10$$

$$15.50 + 50 \leq 135 \quad \text{Multiply 5 and 10.}$$

$$65.50 \leq 135 \quad \text{Add 15.50 and 50.}$$

The estimate is too low. Increase the value of n.

n	$15.50 + 5n \leq 135$	Reasonable?
20	$15.50 + 5(20) \stackrel{?}{\leq} 135 \;\rightarrow\; 115.50 \leq 135$	too low
25	$15.50 + 5(25) \stackrel{?}{\leq} 135 \;\rightarrow\; 140.50 \nleq 135$	too high
23	$15.50 + 5(23) \stackrel{?}{\leq} 135 \;\rightarrow\; 130.50 \leq 135$	almost
24	$15.50 + 5(24) \stackrel{?}{\leq} 135 \;\rightarrow\; 135.50 \leq 135$	too high

Study Tip

Reading Math
In $\{1, 2, 3, 4, \ldots\}$, the three dots are an *ellipsis*. In math, an ellipsis is used to indicate that numbers continue in the same pattern.

Examine   The solution set is $\{0, 1, 2, 3, \ldots, 21, 22, 23\}$. In addition to the first kit, the association can buy as many as 23 additional kits. So, the association can buy as many as $1 + 23$ or 24 garage sale kits and stay within their budget.

Check for Understanding

Concept Check   1. **Describe** the difference between an expression and an open sentence.

2. **OPEN ENDED**   Write an inequality that has a solution set of $\{8, 9, 10, 11, \ldots\}$.

3. **Explain** why an open sentence always has at least one variable.

Guided Practice   Find the solution of each equation if the replacement set is $\{10, 11, 12, 13, 14, 15\}$.

4. $3x - 7 = 29$
5. $12(x - 8) = 84$

Find the solution of each equation using the given replacement set.

6. $x + \dfrac{2}{5} = 1\dfrac{3}{20}; \left\{\dfrac{1}{4}, \dfrac{1}{2}, \dfrac{3}{4}, 1, 1\dfrac{1}{4}\right\}$
7. $7.2(x + 2) = 25.92; \{1.2, 1.4, 1.6, 1.8\}$

Solve each equation.

8. $4(6) + 3 = x$
9. $w = \dfrac{14 - 8}{2}$

Find the solution set for each inequality using the given replacement set.

10. $24 - 2x \geq 13; \{0, 1, 2, 3, 4, 5, 6\}$
11. $3(12 - x) - 2 \leq 28; \{1.5, 2, 2.5, 3\}$

Application   **NUTRITION**   For Exercises 12 and 13, use the following information.
A person must burn 3500 Calories to lose one pound of weight.

12. Write an equation that represents the number of Calories a person would have to burn a day to lose four pounds in two weeks.

13. How many Calories would the person have to burn each day?

Practice and Apply

Find the solution of each equation if the replacement sets are $a = \{0, 3, 5, 8, 10\}$ and $b = \{12, 17, 18, 21, 25\}$.

14. $b - 12 = 9$ **15.** $34 - b = 22$ **16.** $3a + 7 = 31$

17. $4a + 5 = 17$ **18.** $\dfrac{40}{a} - 4 = 0$ **19.** $\dfrac{b}{3} - 2 = 4$

Find the solution of each equation using the given replacement set.

20. $x + \dfrac{7}{4} = \dfrac{17}{8}; \left\{\dfrac{1}{8}, \dfrac{3}{8}, \dfrac{5}{8}, \dfrac{7}{8}\right\}$ **21.** $x + \dfrac{7}{12} = \dfrac{25}{12}; \left\{\dfrac{1}{2}, 1, 1\dfrac{1}{2}, 2\right\}$

22. $\dfrac{2}{5}(x + 1) = \dfrac{8}{15}; \left\{\dfrac{1}{6}, \dfrac{1}{3}, \dfrac{1}{2}, \dfrac{2}{3}\right\}$ **23.** $2.7(x + 5) = 17.28; \{1.2, 1.3, 1.4, 1.5\}$

24. $16(x + 2) = 70.4; \{2.2, 2.4, 2.6, 2.8\}$ **25.** $21(x + 5) = 216.3; \{3.1, 4.2, 5.3, 6.4\}$

MOVIES For Exercises 26–28, use the table and the following information.
The Conkle family is planning to see a movie. There are two adults, a daughter in high school, and two sons in middle school. They do not want to spend more than $30.

26. The movie theater charges the same price for high school and middle school students. Write an inequality to show the cost for the family to go to the movies.

27. How much will it cost for the family to see a matinee?

28. How much will it cost to see an evening show?

Admission Prices		
	Evening	**Matinee**
Adult	$7.50	All Seats $4.50
Student	$4.50	
Child	$4.50	
Senior	$3.50	

Solve each equation.

29. $14.8 - 3.75 = t$ **30.** $a = 32.4 - 18.95$ **31.** $y = \dfrac{12 \cdot 5}{15 - 3}$

32. $g = \dfrac{15 \cdot 6}{16 - 7}$ **33.** $d = \dfrac{7(3) + 3}{4(3 - 1)} + 6$ **34.** $a = \dfrac{4(14 - 1)}{3(6) - 5} + 7$

35. $p = \dfrac{1}{4}[7(2^3) + 4(5^2) - 6(2)]$ **36.** $n = \dfrac{1}{8}[6(3^2) + 2(4^3) - 2(7)]$

Find the solution set for each inequality using the given replacement set.

37. $a - 2 < 6; \{6, 7, 8, 9, 10, 11\}$ **38.** $a + 7 < 22; \{13, 14, 15, 16, 17\}$

39. $\dfrac{a}{5} \geq 2; \{5, 10, 15, 20, 25\}$ **40.** $\dfrac{2a}{4} \leq 8; \{12, 14, 16, 18, 20, 22\}$

41. $4a - 3 \geq 10.6; \{3.2, 3.4, 3.6, 3.8, 4\}$ **42.** $6a - 5 \geq 23.8; \{4.2, 4.5, 4.8, 5.1, 5.4\}$

43. $3a \leq 4; \left\{0, \dfrac{1}{3}, \dfrac{2}{3}, 1, 1\dfrac{1}{3}\right\}$ **44.** $2b < 5; \left\{1, 1\dfrac{1}{2}, 2, 2\dfrac{1}{2}, 3\right\}$

FOOD For Exercises 45 and 46, use the information about food at the left.

45. Write an equation to find the total number of glasses of milk, juice, and soda the average American drinks in a lifetime.

46. How much milk, juice, and soda does the average American drink in a lifetime?

MAIL ORDER For Exercises 47 and 48, use the following information.
Suppose you want to order several sweaters that cost $39.00 each from an online catalog. There is a $10.95 charge for shipping. You have $102.50 to spend.

47. Write an inequality you could use to determine the maximum number of sweaters you can purchase.

48. What is the maximum number of sweaters you can buy?

More About. . .

Food

During a lifetime, the average American drinks 15,579 glasses of milk, 6220 glasses of juice, and 18,995 glasses of soda.
Source: USA TODAY

49. CRITICAL THINKING Describe the solution set for x if $3x \leq 1$.

50. <u>WRITING IN MATH</u> Answer the question that was posed at the beginning of the lesson.

How can you use open sentences to stay within a budget?

Include the following in your answer:
- an explanation of how to use open sentences to stay within a budget, and
- examples of real-world situations in which you would use an inequality and examples where you would use an equation.

51. Find the solution set for $\dfrac{(5 \cdot n)^2 + 5}{(9 \cdot 3^2) - n} < 28$ if the replacement set is $\{5, 7, 9, 11, 13\}$.

  Ⓐ $\{5\}$ Ⓑ $\{5, 7\}$ Ⓒ $\{7\}$ Ⓓ $\{7, 9\}$

52. Which expression has a value of 17?

  Ⓐ $(9 \times 3) - 63 \div 7$ Ⓑ $6(3 + 2) \div (9 - 7)$

  Ⓒ $27 \div 3 + (12 - 4)$ Ⓓ $2[2(6 - 3)] - 5$

Maintain Your Skills

Mixed Review **Write an algebraic expression for each verbal expression. Then evaluate each expression if $r = 2$, $s = 5$, and $t = \dfrac{1}{2}$.** *(Lesson 1-2)*

53. r squared increased by 3 times s

54. t times the sum of four times s and r

55. the sum of r and s times the square of t

56. r to the fifth power decreased by t

Evaluate each expression. *(Lesson 1-2)*

57. $5^3 + 3(4^2)$ **58.** $\dfrac{38 - 12}{2 \cdot 13}$ **59.** $[5(2 + 1)]^4 + 3$

Getting Ready for the Next Lesson **PREREQUISITE SKILL Find each product. Express in simplest form.**
*(To review **multiplying fractions**, see pages 800 and 801.)*

60. $\dfrac{1}{6} \cdot \dfrac{2}{5}$ **61.** $\dfrac{4}{9} \cdot \dfrac{3}{7}$ **62.** $\dfrac{5}{6} \cdot \dfrac{15}{16}$ **63.** $\dfrac{6}{14} \cdot \dfrac{12}{18}$

64. $\dfrac{8}{13} \cdot \dfrac{2}{11}$ **65.** $\dfrac{4}{7} \cdot \dfrac{4}{9}$ **66.** $\dfrac{3}{11} \cdot \dfrac{7}{16}$ **67.** $\dfrac{2}{9} \cdot \dfrac{24}{25}$

Practice Quiz 1   *Lessons 1-1 through 1-3*

Write a verbal expression for each algebraic expression. *(Lesson 1-1)*

1. $x - 20$ **2.** $5n + 2$ **3.** a^3 **4.** $n^4 - 1$

Evaluate each expression. *(Lesson 1-2)*

5. $6(9) - 2(8 + 5)$ **6.** $4[2 + (18 \div 9)^3]$ **7.** $9(3) - 4^2 + 6^2 \div 2$ **8.** $\dfrac{(5 - 2)^2}{3(4 \cdot 2 - 7)}$

9. Evaluate $\dfrac{5a^2 + c - 2}{6 + b}$ if $a = 4$, $b = 5$, and $c = 10$. *(Lesson 1-2)*

10. Find the solution set for $2n^2 + 3 \leq 75$ if the replacement set is $\{4, 5, 6, 7, 8, 9\}$. *(Lesson 1-3)*

CONTENTS

Identity and Equality Properties

What You'll Learn

- Recognize the properties of identity and equality.
- Use the properties of identity and equality.

Vocabulary
- additive identity
- multiplicative identity
- multiplicative inverses
- reciprocal

How are identity and equality properties used to compare data?

During the college football season, teams are ranked weekly. The table shows the last three rankings of the top five teams for the 2000 football season. The open sentence below represents the change in rank of Oregon State from December 11 to the final rank.

	Dec. 4	Dec. 11	Final Rank
University of Oklahoma	1	1	1
University of Miami	2	2	2
University of Washington	4	3	3
Oregon State University	5	4	4
Florida State University	3	5	5
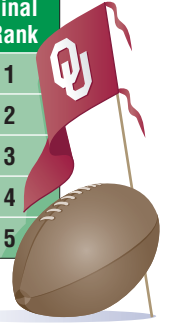
Rank on December 11, 2000	plus	increase in rank	equals	final rank for 2000 season.
4	+	r	=	4

The solution of this equation is 0. Oregon State's rank changed by 0 from December 11 to the final rank. In other words, $4 + 0 = 4$.

IDENTITY AND EQUALITY PROPERTIES The sum of any number and 0 is equal to the number. Thus, 0 is called the **additive identity**.

Key Concept — Additive Identity

- **Words** For any number a, the sum of a and 0 is a.
- **Symbols** $a + 0 = 0 + a = a$
- **Examples** $5 + 0 = 5$, $0 + 5 = 5$

There are also special properties associated with multiplication. Consider the following equations.

$$7 \cdot n = 7$$

The solution of the equation is 1. Since the product of any number and 1 is equal to the number, 1 is called the **multiplicative identity**.

$$9 \cdot m = 0$$

The solution of the equation is 0. The product of any number and 0 is equal to 0. This is called the **Multiplicative Property of Zero**.

$$\frac{1}{3} \cdot 3 = 1$$

Two numbers whose product is 1 are called **multiplicative inverses** or **reciprocals**. Zero has no reciprocal because any number times 0 is 0.

CONTENTS

Key Concept — Multiplication Properties

Property	Words	Symbols	Examples
Multiplicative Identity	For any number a, the product of a and 1 is a.	$a \cdot 1 = 1 \cdot a = a$	$12 \cdot 1 = 12$, $1 \cdot 12 = 12$
Multiplicative Property of Zero	For any number a, the product of a and 0 is 0.	$a \cdot 0 = 0 \cdot a = 0$	$8 \cdot 0 = 0$, $0 \cdot 8 = 0$
Multiplicative Inverse	For every number $\frac{a}{b}$, where $a, b \neq 0$, there is exactly one number $\frac{b}{a}$ such that the product of $\frac{a}{b}$ and $\frac{b}{a}$ is 1.	$\frac{a}{b} \cdot \frac{b}{a} = \frac{b}{a} \cdot \frac{a}{b} = 1$	$\frac{2}{3} \cdot \frac{3}{2} = \frac{6}{6} = 1$, $\frac{3}{2} \cdot \frac{2}{3} = \frac{6}{6} = 1$

Example 1 Identify Properties

Name the property used in each equation. Then find the value of n.

a. $42 \cdot n = 42$

Multiplicative Identity Property

$n = 1$, since $42 \cdot 1 = 42$.

b. $n + 0 = 15$

Additive Identity Property

$n = 15$, since $15 + 0 = 15$.

c. $n \cdot 9 = 1$

Multiplicative Inverse Property

$n = \frac{1}{9}$, since $\frac{1}{9} \cdot 9 = 1$.

There are several properties of equality that apply to addition and multiplication. These are summarized below.

Key Concept — Properties of Equality

Property	Words	Symbols	Examples
Reflexive	Any quantity is equal to itself.	For any number a, $a = a$.	$7 = 7$, $2 + 3 = 2 + 3$
Symmetric	If one quantity equals a second quantity, then the second quantity equals the first.	For any numbers a and b, if $a = b$, then $b = a$.	If $9 = 6 + 3$, then $6 + 3 = 9$.
Transitive	If one quantity equals a second quantity and the second quantity equals a third quantity, then the first quantity equals the third quantity.	For any numbers a, b, and c, if $a = b$ and $b = c$, then $a = c$.	If $5 + 7 = 8 + 4$ and $8 + 4 = 12$, then $5 + 7 = 12$.
Substitution	A quantity may be substituted for its equal in any expression.	If $a = b$, then a may be replaced by b in any expression.	If $n = 15$, then $3n = 3 \cdot 15$.

CONTENTS

USE IDENTITY AND EQUALITY PROPERTIES The properties of identity and equality can be used to justify each step when evaluating an expression.

Example 2 **Evaluate Using Properties**

Evaluate $2(3 \cdot 2 - 5) + 3 \cdot \frac{1}{3}$. Name the property used in each step.

$$2(3 \cdot 2 - 5) + 3 \cdot \frac{1}{3} = 2(6 - 5) + 3 \cdot \frac{1}{3} \qquad \text{Substitution; } 3 \cdot 2 = 6$$

$$= 2(1) + 3 \cdot \frac{1}{3} \qquad \text{Substitution; } 6 - 5 = 1$$

$$= 2 + 3 \cdot \frac{1}{3} \qquad \text{Multiplicative Identity; } 2 \cdot 1 = 2$$

$$= 2 + 1 \qquad \text{Multiplicative Inverse; } 3 \cdot \frac{1}{3} = 1$$

$$= 3 \qquad \text{Substitution; } 2 + 1 = 3$$

Check for Understanding

Concept Check
1. **Explain** whether 1 can be an additive identity.

2. **OPEN ENDED** Write two equations demonstrating the Transitive Property of Equality.

3. **Explain** why 0 has no multiplicative inverse.

Guided Practice Name the property used in each equation. Then find the value of n.

4. $13n = 0$
5. $17 + 0 = n$
6. $\frac{1}{6}n = 1$

7. Evaluate $6(12 - 48 \div 4)$. Name the property used in each step.

8. Evaluate $\left(15 \cdot \frac{1}{15} + 8 \cdot 0\right) \cdot 12$. Name the property used in each step.

Application **HISTORY** For Exercises 9–11, use the following information.
On November 19, 1863, Abraham Lincoln delivered the famous Gettysburg Address. The speech began "Four score and seven years ago, . . ."

9. Write an expression to represent four score and seven. (*Hint*: A score is 20.)

10. Evaluate the expression. Name the property used in each step.

11. How many years is four score and seven?

Practice and Apply

Name the property used in each equation. Then find the value of n.

12. $12n = 12$
13. $n \cdot 1 = 5$
14. $8 \cdot n = 8 \cdot 5$

15. $0.25 + 1.5 = n + 1.5$
16. $8 = n + 8$
17. $n + 0 = \frac{1}{3}$

18. $1 = 2n$
19. $4 \cdot \frac{1}{4} = n$
20. $(9 - 7)(5) = 2(n)$

21. $3 + (2 + 8) = n + 10$
22. $n\left(5^2 \cdot \frac{1}{25}\right) = 3$
23. $6\left(\frac{1}{2} \cdot n\right) = 6$

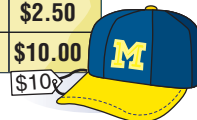Evaluate each expression. Name the property used in each step.

24. $\frac{3}{4}[4 \div (7 - 4)]$
25. $\frac{2}{3}[3 \div (2 \cdot 1)]$
26. $2(3 \cdot 2 - 5) + 3 \cdot \frac{1}{3}$

27. $6 \cdot \frac{1}{6} + 5(12 \div 4 - 3)$
28. $3 + 5(4 - 2^2) - 1$
29. $7 - 8(9 - 3^2)$

FUND-RAISING For Exercises 30 and 31, use the following information.
The spirit club at Central High School is selling items to raise money. The profit the club earns on each item is the difference between what an item sells for and what it costs the club to buy.

School Spirit Items		
Item	**Cost**	**Selling Price**
Pennant	$3.00	$5.00
Button	$1.00	$2.50
Cap	$6.00	$10.00

30. Write an expression that represents the profit for 25 pennants, 80 buttons, and 40 caps.

31. Evaluate the expression, indicating the property used in each step.

MILITARY PAY For Exercises 32 and 33, use the table that shows the monthly base pay rates for the first five ranks of enlisted personnel.

Monthly Basic Pay Rates by Grade, Effective July 1, 2001								
Years of Service								
Grade	< 2	> 2	> 3	> 4	> 6	> 8	> 10	> 12
E-5	1381.80	1549.20	1623.90	1701.00	1779.30	1888.50	1962.90	2040.30
E-4	1288.80	1423.80	1500.60	1576.20	1653.00	1653.00	1653.00	1653.00
E-3	1214.70	1307.10	1383.60	1385.40	1385.40	1385.40	1385.40	1385.40
E-2	1169.10	1169.10	1169.10	1169.10	1169.10	1169.10	1169.10	1169.10
E-1	1042.80	1042.80	1042.80	1042.80	1042.80	1042.80	1042.80	1042.80

Source: U.S. Department of Defense

32. Write an equation using addition that shows the change in pay for an enlisted member at grade E-2 from 3 years of service to 12 years.

33. Write an equation using multiplication that shows the change in pay for someone at grade E-4 from 6 years of service to 10 years.

FOOTBALL For Exercises 34–36, use the table that shows the base salary and various bonus plans for the NFL from 2002–2005.

34. Suppose a player rushed for 12 touchdowns in 2002 and another player scored 76 points that same year. Write an equation that compares the two salaries and bonuses.

35. Write an expression that could be used to determine what a team owner would pay in base salaries and bonuses in 2004 for the following:
   - eight players who keep their weight under 240 pounds and are involved in at least 35% of the offensive plays,
   - three players who score 12 rushing touchdowns and score 76 points, and
   - four players who gain 1601 yards of total offense and average 4.5 yards per carry.

36. Evaluate the expression you wrote in Exercise 35. Name the property used in each step.

NFL Salaries and Bonuses	
Year	**Base Salary**
2002	$350,000
2003	375,000
2004	400,000
2005	400,000
Goal	**Bonus**
Involved in 35% of offensive plays	$50,000
Average 4.5 yards per carry	50,000
12 rushing touchdowns	50,000
12 receiving touchdowns	50,000
76 points scored	50,000
1601 yards of total offense	50,000
Keep weight below 240 lb	100,000
Goal—Rushing Yards	**Bonus**
1600 yards	$1 million
1800 yards	1.5 million
2000 yards	2 million
2100 yards	2.5 million

Source: ESPN Sports Almanac

Online Research **Data Update** Find the most recent statistics for a professional football player. What were his base salary and bonuses? Visit www.algebra1.com/data_update to learn more.

CONTENTS

37. CRITICAL THINKING The Transitive Property of Inequality states that if $a < b$ and $b < c$, then $a < c$. Use this property to determine whether the following statement is *sometimes*, *always*, or *never* true.

$$\text{If } x > y \text{ and } z > w, \text{ then } xz > yw.$$

Give examples to support your answer.

38. <u>WRITING IN MATH</u>   Answer the question that was posed at the beginning of the lesson.

How are identity and equality properties used to compare data?

Include the following in your answer:

- a description of how you could use the Reflexive or Symmetric Property to compare a team's rank for any two time periods, and
- a demonstration of the Transitive Property using one of the team's three rankings as an example.
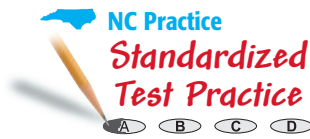
39. Which equation illustrates the Symmetric Property of Equality?

Ⓐ If $a = b$, then $b = a$.
Ⓑ If $a = b$, $b = c$, then $a = c$.
Ⓒ If $a = b$, then $b = c$.
Ⓓ If $a = a$, then $a + 0 = a$.

40. The equation $(10 - 8)(5) = (2)(5)$ is an example of which property of equality?

Ⓐ Reflexive
Ⓑ Substitution
Ⓒ Symmetric
Ⓓ Transitive

Extending the Lesson   The sum of any two whole numbers is always a whole number. So, the set of whole numbers $\{0, 1, 2, 3, …\}$ is said to be closed under addition. This is an example of the ==Closure Property==. State whether each of the following statements is *true* or *false*. If false, justify your reasoning.

41. The set of whole numbers is closed under subtraction.

42. The set of whole numbers is closed under multiplication.

43. The set of whole numbers is closed under division.

Maintain Your Skills

Mixed Review   **Find the solution set for each inequality using the given replacement set.**
(Lesson 1-3)

44. $10 - x > 6$; $\{3, 5, 6, 8\}$

45. $4x + 2 < 58$; $\{11, 12, 13, 14, 15\}$

46. $\frac{x}{2} \geq 3$; $\{5.8, 5.9, 6, 6.1, 6.2, 6.3\}$

47. $8x \leq 32$; $\{3, 3.25, 3.5, 3.75, 4\}$

48. $\frac{7}{10} - 2x < \frac{3}{10}$; $\left\{\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}\right\}$

49. $2x - 1 \leq 2$; $\left\{1\frac{1}{4}, 2, 3, 3\frac{1}{2}\right\}$

Evaluate each expression.   *(Lesson 1-2)*

50. $(3 + 9) \div 3^2$

51. $6(12 - 7.5) - 7$

52. $20 \div 4 \cdot 8 \div 10$

53. $\frac{(6 + 2)^2}{16} + 3(9)$

54. $[6^2 - (2 + 4)2]3$

55. $9(3) - 4^2 + 6^2 \div 2$

56. Write an algebraic expression for the sum of twice a number squared and 7.   *(Lesson 1-1)*

Getting Ready for the Next Lesson   **PREREQUISITE SKILL**   **Evaluate each expression.**
*(To review **order of operations**, see Lesson 1-2.)*

57. $10(6) + 10(2)$

58. $(15 - 6) \cdot 8$

59. $12(4) - 5(4)$

60. $3(4 + 2)$

61. $5(6 - 4)$

62. $8(14 + 2)$

The Distributive Property

Vocabulary

- term
- like terms
- equivalent expressions
- simplest form
- coefficient

What You'll Learn

- Use the Distributive Property to evaluate expressions.
- Use the Distributive Property to simplify expressions.

How can the Distributive Property be used to calculate quickly?

Instant Replay Video Games sells new and used games. During a Saturday morning sale, the first 8 customers each bought a bargain game and a new release. To calculate the total sales for these customers, you can use the Distributive Property.

Sale Prices	
Used Games	$9.95
Bargain Games	$14.95
Regular Games	$24.95
New Releases	$34.95

EVALUATE EXPRESSIONS There are two methods you could use to calculate the video game sales.

Method 1

sales of bargain games	plus	sales of new releases
8(14.95)	+	8(34.95)

$$= 119.60 + 279.60$$
$$= 399.20$$

Method 2

number of customers	times	each customer's purchase price
8	×	(14.95 + 34.95)

$$= 8(49.90)$$
$$= 399.20$$

Either method gives total sales of $399.20 because the following is true.

$$8(14.95) + 8(34.95) = 8(14.95 + 34.95)$$

This is an example of the **Distributive Property**.

Key Concept   *Distributive Property*

- **Symbols** For any numbers a, b, and c,
  $a(b + c) = ab + ac$ and $(b + c)a = ba + ca$ and
  $a(b - c) = ab - ac$ and $(b - c)a = ba - ca$.

- **Examples** $3(2 + 5) = 3 \cdot 2 + 3 \cdot 5$ $4(9 - 7) = 4 \cdot 9 - 4 \cdot 7$
  $3(7) = 6 + 15$ $4(2) = 36 - 28$
  $21 = 21$ ✓ $8 = 8$ ✓

Notice that it does not matter whether a is placed on the right or the left of the expression in the parentheses.

The Symmetric Property of Equality allows the Distributive Property to be written as follows.

If $a(b + c) = ab + ac$, then $ab + ac = a(b + c)$.

Example 1 Distribute Over Addition

Rewrite 8(10 + 4) using the Distributive Property. Then evaluate.

$8(10 + 4) = 8(10) + 8(4)$ Distributive Property

$\qquad\qquad = 80 + 32$ Multiply.

$\qquad\qquad = 112$ Add.

Example 2 Distribute Over Subtraction

Rewrite (12 − 3)6 using the Distributive Property. Then evaluate.

$(12 - 3)6 = 12 \cdot 6 - 3 \cdot 6$ Distributive Property

$\qquad\qquad = 72 - 18$ Multiply.

$\qquad\qquad = 54$ Subtract.

Example 3 Use the Distributive Property

CARS The Morris family owns two cars. In 1998, they drove the first car 18,000 miles and the second car 16,000 miles. Use the graph to find the total cost of operating both cars.

Use the Distributive Property to write and evaluate an expression.

$0.46(18,000 + 16,000)$ Distributive Prop.

$\qquad = 8280 + 7360$ Multiply.

$\qquad = 15,640$ Add.

It cost the Morris family $15,640 to operate their cars.

USA TODAY Snapshots®

Car costs race ahead

The average cents-per-mile cost of owning and operating an automobile in the USA, by year:

1985 **23¢**

1990 **33¢**

1995 **41¢**

1998 **46¢**
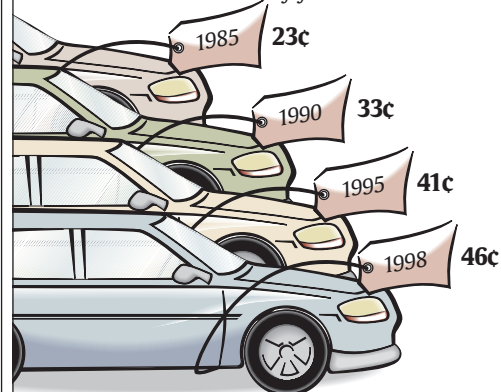
Source: Transportation Department; American Automobile Association

By Marcy E. Mullins, USA TODAY

The Distributive Property can be used to simplify mental calculations.

Example 4 Use the Distributive Property

Use the Distributive Property to find each product.

a. 15 · 99

$15 \cdot 99 = 15(100 - 1)$ Think: $99 = 100 - 1$

$\qquad\quad = 15(100) - 15(1)$ Distributive Property

$\qquad\quad = 1500 - 15$ Multiply.

$\qquad\quad = 1485$ Subtract.

b. $35\left(2\frac{1}{5}\right)$

$35\left(2\frac{1}{5}\right) = 35\left(2 + \frac{1}{5}\right)$ Think: $2\frac{1}{5} = 2 + \frac{1}{5}$

$\qquad\quad = 35(2) + 35\left(\frac{1}{5}\right)$ Distributive Property

$\qquad\quad = 70 + 7$ Multiply.

$\qquad\quad = 77$ Add.

www.algebra1.com/extra_examples/nc

CONTENTS

SIMPLIFY EXPRESSIONS   You can use algebra tiles to investigate how the Distributive Property relates to algebraic expressions.
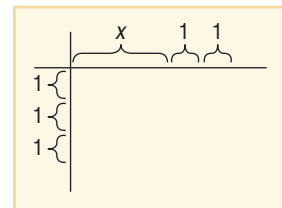
Algebra Activity

The Distributive Property

Consider the product $3(x + 2)$. Use a product mat and algebra tiles to model $3(x + 2)$ as the area of a rectangle whose dimensions are 3 and $(x + 2)$.

Step 1   Use algebra tiles to mark the dimensions of the rectangle on a product mat.

Step 2   Using the marks as a guide, make the rectangle with the algebra tiles. The rectangle has 3 x-tiles and 6 1-tiles. The area of the rectangle is $x + 1 + 1 + x + 1 + 1 + x + 1 + 1$ or $3x + 6$. Therefore, $3(x + 2) = 3x + 6$.
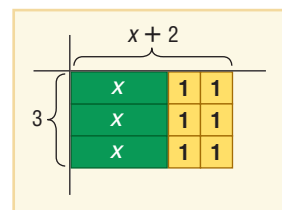
Model and Analyze

Find each product by using algebra tiles.

1. $2(x + 1)$ **2.** $5(x + 2)$ **3.** $2(2x + 1)$

Tell whether each statement is *true* or *false*. Justify your answer with algebra tiles and a drawing.

4. $3(x + 3) = 3x + 3$ **5.** $x(3 + 2) = 3x + 2x$

6. Rachel says that $3(x + 4) = 3x + 12$, but José says that $3(x + 4) = 3x + 4$. Use words and models to explain who is correct and why.

You can apply the Distributive Property to algebraic expressions.

Example 5   Algebraic Expressions

Rewrite each product using the Distributive Property. Then simplify.

a. $5(g - 9)$

$5(g - 9) = 5 \cdot g - 5 \cdot 9$ Distributive Property

$\qquad\quad = 5g - 45$ Multiply.

b. $3(2x^2 + 4x - 1)$

$3(2x^2 + 4x - 1) = (3)(2x^2) + (3)(4x) - 3(1)$ Distributive Property

$\qquad\qquad\qquad\quad = 6x^2 + 12x - 3$ Simplify.

A **term** is a number, a variable, or a product or quotient of numbers and variables. For example, y, p^3, $4a$, and $5g^2h$ are all terms. **Like terms** are terms that contain the same variables, with corresponding variables having the same power.

$$2x^2 + 6x + 5 \qquad\qquad 3a^2 + 5a^2 + 2a$$

three terms   like terms   unlike terms

CONTENTS

The Distributive Property and the properties of equality can be used to show that $5n + 7n = 12n$. In this expression, $5n$ and $7n$ are like terms.

$$5n + 7n = (5 + 7)n \quad \text{Distributive Property}$$
$$= 12n \quad \text{Substitution}$$

The expressions $5n + 7n$ and $12n$ are called **equivalent expressions** because they denote the same number. An expression is in **simplest form** when it is replaced by an equivalent expression having no like terms or parentheses.

Example 6 Combine Like Terms

Simplify each expression.

a. $15x + 18x$

$$15x + 18x = (15 + 18)x \quad \text{Distributive Property}$$
$$= 33x \quad \text{Substitution}$$

b. $10n + 3n^2 + 9n^2$

$$10n + 3n^2 + 9n^2 = 10n + (3 + 9)n^2 \quad \text{Distributive Property}$$
$$= 10n + 12n^2 \quad \text{Substitution}$$

The **coefficient** of a term is the numerical factor. For example, in $17xy$, the coefficient is 17, and in $\dfrac{3y^2}{4}$, the coefficient is $\dfrac{3}{4}$. In the term m, the coefficient is 1 since $1 \cdot m = m$ by the Multiplicative Identity Property.

Check for Understanding

Concept Check

1. **Explain** why the Distributive Property is sometimes called The Distributive Property of Multiplication Over Addition.

2. **OPEN ENDED** Write an expression that has five terms, three of which are like terms and one term with a coefficient of 1.

3. **FIND THE ERROR** Courtney and Ben are simplifying $4w^4 + w^4 + 3w^2 - 2w^2$.

Courtney	Ben
$4w^4 + w^4 + 3w^2 - 2w^2$	$4w^4 + w^4 + 3w^2 - 2w^2$
$= (4 + 1)w^4 + (3 - 2)w^2$	$= (4)w^4 + (3 - 2)w^2$
$= 5w^4 + 1w^2$	$= 4w^4 + 1w^2$
$= 5w^4 + w^2$	$= 4w^4 + w^2$

Who is correct? Explain your reasoning.

Guided Practice

Rewrite each expression using the Distributive Property. Then simplify.

4. $6(12 - 2)$ 5. $2(4 + t)$ 6. $(g - 9)5$

Use the Distributive Property to find each product.

7. $16(102)$ 8. $\left(3\frac{1}{17}\right)(17)$

Simplify each expression. If not possible, write *simplified*.

9. $13m + m$ 10. $3(x + 2x)$

11. $14a^2 + 13b^2 + 27$ 12. $4(3g + 2)$

CONTENTS

COSMETOLOGY For Exercises 13 and 14, use the following information.
Ms. Curry owns a hair salon. One day, she gave 12 haircuts. She earned $19.95 for each and received an average tip of $2 for each haircut.

13. Write an expression to determine the total amount she earned.

14. How much did Ms. Curry earn?

Practice and Apply

Rewrite each expression using the Distributive Property. Then simplify.

15. $8(5 + 7)$
16. $7(13 + 12)$
17. $12(9 - 5)$
18. $13(10 - 7)$
19. $3(2x + 6)$
20. $8(3m + 4)$
21. $(4 + x)2$
22. $(5 + n)3$
23. $28\left(y - \frac{1}{7}\right)$
24. $27\left(2b - \frac{1}{3}\right)$
25. $a(b - 6)$
26. $x(z + 3)$
27. $2(a - 3b + 2c)$
28. $4(8p + 4q - 7r)$

OLYMPICS For Exercises 29 and 30, use the following information.
At the 2000 Summer Olympics in Australia, about 110,000 people attended events at Olympic Stadium each day while another 17,500 fans were at the aquatics center.

29. Write an expression you could use to determine the total number of people at Olympic Stadium and the Aquatic Center over 4 days.

30. What was the attendance for the 4-day period?

Use the Distributive Property to find each product.

31. $5 \cdot 97$
32. $8 \cdot 990$
33. $17 \cdot 6$
34. $24 \cdot 7$
35. $18\left(2\frac{1}{9}\right)$
36. $48\left(3\frac{1}{6}\right)$

COMMUNICATIONS For Exercises 37 and 38, use the following information.
A public relations consultant keeps a log of all contacts made by e-mail, telephone, and in person. In a typical week, she averages 5 hours using e-mail, 12 hours of meeting in person, and 18 hours on the telephone.

37. Write an expression that could be used to predict how many hours she will spend on these activities over the next 12 weeks.

38. How many hours should she plan for contacting people for the next 12 weeks?

INSURANCE For Exercises 39–41, use the table that shows the monthly cost of a company health plan.

Available Insurance Plans—Monthly Charge			
Coverage	Medical	Dental	Vision
Employee	$78	$20	$12
Family (additional coverage)	$50	$15	$7

39. Write an expression that could be used to calculate the cost of medical, dental, and vision insurance for an employee for 6 months.

40. How much does it cost an employee to get all three types of insurance for 6 months?

41. How much would an employee expect to pay for individual and family medical and dental coverage per year?

Simplify each expression. If not possible, write *simplified*.

42. $2x + 9x$

43. $4b + 5b$

44. $5n^2 + 7n$

45. $3a^2 + 14a^2$

46. $12(3c + 4)$

47. $15(3x - 5)$

48. $6x^2 + 14x - 9x$

49. $4y^3 + 3y^3 + y^4$

50. $6(5a + 3b - 2b)$

51. $5(6m + 4n - 3n)$

52. $x^2 + \frac{7}{8}x - \frac{x}{8}$

**53.** $a + \frac{a}{5} + \frac{2}{5}a$

54. CRITICAL THINKING The expression $2(\ell + w)$ may be used to find the perimeter of a rectangle. What are the length and width of a rectangle if the area is $13\frac{1}{2}$ square units and the length of one side is $\frac{1}{5}$ the measure of the perimeter?

55. WRITING IN MATH Answer the question that was posed at the beginning of the lesson.

How can the Distributive Property be used to calculate quickly?

Include the following in your answer:

- a comparison of the two methods of finding the total video game sales.
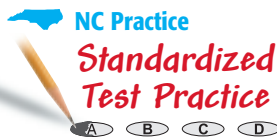
56. Simplify $3(x + y) + 2(x + y) - 4x$.

Ⓐ $5x + y$ 　　Ⓑ $9x + 5y$ 　　Ⓒ $5x + 9y$ 　　Ⓓ $x + 5y$

57. If $a = 2.8$ and $b = 4.2$, find the value of c in the equation $c = 7(2a + 3b)$.

Ⓐ 18.2 　　Ⓑ 238.0 　　Ⓒ 127.4 　　Ⓓ 51.8

Maintain Your Skills

Mixed Review **Name the property illustrated by each statement or equation.** *(Lesson 1-4)*

58. If $7 \cdot 2 = 14$, then $14 = 7 \cdot 2$.

59. $8 + (3 + 9) = 8 + 12$

60. $mnp = 1mnp$

61. $3\left(5^2 \cdot \frac{1}{25}\right) = 3 \cdot 1$

62. $\left(\frac{3}{4}\right)\left(\frac{4}{3}\right) = 1$

63. $32 + 21 = 32 + 21$

PHYSICAL SCIENCE **For Exercises 64 and 65, use the following information.**
Sound travels 1129 feet per second through air. *(Lesson 1-3)*

64. Write an equation that represents how many feet sound can travel in 2 seconds when it is traveling through air.

65. How far can sound travel in 2 seconds when traveling through air?
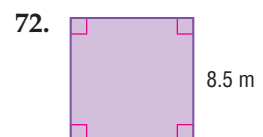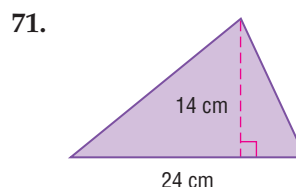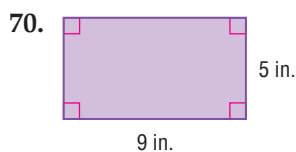
Evaluate each expression if $a = 4$, $b = 6$, and $c = 3$. *(Lesson 1–2)*

66. $3ab - c^2$ 　　**67.** $8(a - c)^2 + 3$ 　　**68.** $\dfrac{6ab}{c(a + 2)}$ 　　**69.** $(a + c)\left(\dfrac{a + b}{2}\right)$

Getting Ready for the Next Lesson **PREREQUISITE SKILL** **Find the area of each figure.**
*(To review **finding area**, see pages 813 and 814.)*

70.

5 in.

9 in.

71.

14 cm

24 cm

72.

8.5 m

Commutative and Associative Properties

What You'll Learn

• Recognize the Commutative and Associative Properties.

• Use the Commutative and Associative Properties to simplify expressions.

How can properties help you determine distances?

The South Line of the Atlanta subway leaves Five Points and heads for Garnett, 0.4 mile away. From Garnett, West End is 1.5 miles. The distance from Five Points to West End can be found by evaluating the expression 0.4 + 1.5. Likewise, the distance from West End to Five Points can be found by evaluating the expression 1.5 + 0.4.

Five Points
0.4 mi — S1 Garnett
1.5 mi
S2 West End
1.5 mi
S3 Oakland City
1.1 mi
S4 Lakewood/Ft. McPherson
1.9 mi
S5 East Point
1.8 mi
S6 College Park
0.8 mi
S7 Airport
South Line

COMMUTATIVE AND ASSOCIATIVE PROPERTIES In the situation above, the distance from Five Points to West End is the same as the distance from West End to Five Points. This distance can be represented by the following equation.

The distance from Five Points to West End	equals	the distance from West End to Five Points.
$0.4 + 1.5$	$=$	$1.5 + 0.4$

This is an example of the **Commutative Property**.

Key Concept — Commutative Property

• **Words** The order in which you add or multiply numbers does not change their sum or product.

• **Symbols** For any numbers a and b, $a + b = b + a$ and $a \cdot b = b \cdot a$.

• **Examples** $5 + 6 = 6 + 5$, $3 \cdot 2 = 2 \cdot 3$

An easy way to find the sum or product of numbers is to group, or associate, the numbers using the **Associative Property**.

Key Concept — Associative Property

• **Words** The way you group three or more numbers when adding or multiplying does not change their sum or product.

• **Symbols** For any numbers a, b, and c,
$(a + b) + c = a + (b + c)$ and $(ab)c = a(bc)$.

• **Examples** $(2 + 4) + 6 = 2 + (4 + 6)$, $(3 \cdot 5) \cdot 4 = 3 \cdot (5 \cdot 4)$

CONTENTS

Example 1 Multiplication Properties

Evaluate $8 \cdot 2 \cdot 3 \cdot 5$.

You can rearrange and group the factors to make mental calculations easier.

$$
\begin{aligned}
8 \cdot 2 \cdot 3 \cdot 5 &= 8 \cdot 3 \cdot 2 \cdot 5 && \text{Commutative } (\times) \\
&= (8 \cdot 3) \cdot (2 \cdot 5) && \text{Associative } (\times) \\
&= 24 \cdot 10 && \text{Multiply.} \\
&= 240 && \text{Multiply.}
\end{aligned}
$$

Example 2 Use Addition Properties

TRANSPORTATION   Refer to the application at the beginning of the lesson. Find the distance between Five Points and Lakewood/Ft. McPherson.

Five Points to Garnett		Garnett to West End		West End to Oakland City		Oakland City to Lakewood/Ft. McPherson
0.4	+	1.5	+	1.5	+	1.1

$$
\begin{aligned}
0.4 + 1.5 + 1.5 + 1.1 &= 0.4 + 1.1 + 1.5 + 1.5 && \text{Commutative } (+) \\
&= (0.4 + 1.1) + (1.5 + 1.5) && \text{Associative } (+) \\
&= 1.5 + 3.0 && \text{Add.} \\
&= 4.5 && \text{Add.}
\end{aligned}
$$

Lakewood/Ft. McPherson is 4.5 miles from Five Points.

SIMPLIFY EXPRESSIONS   The Commutative and Associative Properties can be used with other properties when evaluating and simplifying expressions.

More About . . .

Transportation

New York City has the most extensive subway system, covering 842 miles of track and serving about 4.3 million passengers per day.

Source: *The Guinness Book of Records*

Concept Summary		Properties of Numbers

The following properties are true for any numbers a, b, and c.

Properties	Addition	Multiplication
Commutative	$a + b = b + a$	$ab = ba$
Associative	$(a + b) + c = a + (b + c)$	$(ab)c = a(bc)$
Identity	0 is the identity. $a + 0 = 0 + a = a$	1 is the identity. $a \cdot 1 = 1 \cdot a = a$
Zero	——	$a \cdot 0 = 0 \cdot a = 0$
Distributive	$a(b + c) = ab + ac$ and $(b + c)a = ba + ca$	
Substitution	If $a = b$, then a may be substituted for b.	

Example 3 Simplify an Expression

Simplify $3c + 5(2 + c)$.

$$
\begin{aligned}
3c + 5(2 + c) &= 3c + 5(2) + 5(c) && \text{Distributive Property} \\
&= 3c + 10 + 5c && \text{Multiply.} \\
&= 3c + 5c + 10 && \text{Commutative } (+) \\
&= (3c + 5c) + 10 && \text{Associative } (+) \\
&= (3 + 5)c + 10 && \text{Distributive Property} \\
&= 8c + 10 && \text{Substitution}
\end{aligned}
$$

www.algebra1.com/extra_examples/nc

CONTENTS

Example 4 Write and Simplify an Expression

Use the expression *four times the sum of a and b increased by twice the sum of a and 2b.*

a. Write an algebraic expression for the verbal expression.

$$\underbrace{\text{four times the}}_{4(a+b)}\text{ sum of } a \text{ and } b \quad \underbrace{\text{increased by}}_{+} \quad \underbrace{\text{twice the sum}}_{2(a+2b)}\text{ of } a \text{ and } 2b$$

b. Simplify the expression and indicate the properties used.

$4(a+b) + 2(a+2b) = 4(a) + 4(b) + 2(a) + 2(2b)$	Distributive Property
$= 4a + 4b + 2a + 4b$	Multiply.
$= 4a + 2a + 4b + 4b$	Commutative ($+$)
$= (4a + 2a) + (4b + 4b)$	Associative ($+$)
$= (4 + 2)a + (4 + 4)b$	Distributive Property
$= 6a + 8b$	Substitution

Check for Understanding

Concept Check
1. **Define** the Associative Property in your own words.
2. **Write** a short explanation as to whether there is a Commutative Property of Division.
3. **OPEN ENDED** Write examples of the Commutative Property of Addition and the Associative Property of Multiplication using 1, 5, and 8 in each.

Guided Practice Evaluate each expression.
4. $14 + 18 + 26$
5. $3\frac{1}{2} + 4 + 2\frac{1}{2}$
6. $5 \cdot 3 \cdot 6 \cdot 4$
7. $\frac{5}{6} \cdot 16 \cdot 9\frac{3}{4}$

Simplify each expression.
8. $4x + 5y + 6x$
9. $5a + 3b + 2a + 7b$
10. $\frac{1}{4}q + 2q + 2\frac{3}{4}q$
11. $3(4x + 2) + 2x$
12. $7(ac + 2b) + 2ac$
13. $3(x + 2y) + 4(3x + y)$

14. Write an algebraic expression for *half the sum of p and 2q increased by three-fourths q.* Then simplify, indicating the properties used.

Application
15. **GEOMETRY** Find the area of the large triangle if each smaller triangle has a base measuring 5.2 centimeters and a height of 7.86 centimeters.
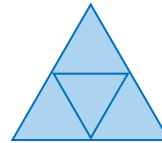
Practice and Apply

Evaluate each expression.
16. $17 + 6 + 13 + 24$
17. $8 + 14 + 22 + 9$
18. $4.25 + 3.50 + 8.25$
19. $6.2 + 4.2 + 4.3 + 5.8$
20. $6\frac{1}{2} + 3 + \frac{1}{2} + 2$
21. $2\frac{3}{8} + 4 + 3\frac{3}{8}$
22. $5 \cdot 11 \cdot 4 \cdot 2$
23. $3 \cdot 10 \cdot 6 \cdot 3$
24. $0.5 \cdot 2.4 \cdot 4$
25. $8 \cdot 1.6 \cdot 2.5$
26. $3\frac{3}{7} \cdot 14 \cdot 1\frac{1}{4}$
27. $2\frac{5}{8} \cdot 24 \cdot 6\frac{2}{3}$

CONTENTS
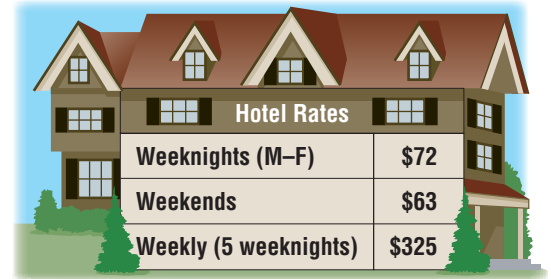
TRAVEL For Exercises 28 and 29, use the following information.
Hotels often have different rates for weeknights and weekends. The rates of one hotel are listed in the table.

28. If a traveler checks into the hotel on Friday and checks out the following Tuesday morning, what is the total cost of the room?

29. Suppose there is a sales tax of $5.40 for weeknights and $5.10 for weekends. What is the total cost of the room including tax?

Hotel Rates	
Weeknights (M–F)	$72
Weekends	$63
Weekly (5 weeknights)	$325

ENTERTAINMENT For Exercises 30 and 31, use the following information.
A video store rents new release videos for $4.49, older videos for $2.99, and DVDs for $3.99. The store also sells its used videos for $9.99.

30. Write two expressions to represent the total sales of a clerk after renting 2 DVDs, 3 new releases, 2 older videos, and selling 5 used videos.

31. What are the total sales of the clerk?

Simplify each expression.

32. $4a + 2b + a$

33. $2y + 2x + 8y$

34. $x^2 + 3x + 2x + 5x^2$

35. $4a^3 + 6a + 3a^3 + 8a$

36. $6x + 2(2x + 7)$

37. $5n + 4(3n + 9)$

38. $3(x + 2y) + 4(3x + y)$

39. $3.2(x + y) + 2.3(x + y) + 4x$

40. $3(4m + n) + 2m$

41. $6(0.4f + 0.2g) + 0.5f$

42. $\frac{3}{4} + \frac{2}{3}(s + 2t) + s$

43. $2p + \frac{3}{5}\left(\frac{1}{2}p + 2q\right) + \frac{2}{3}$

Write an algebraic expression for each verbal expression. Then simplify, indicating the properties used.

44. twice the sum of s and t decreased by s

45. five times the product of x and y increased by $3xy$

46. the product of six and the square of z, increased by the sum of seven, z^2, and 6

47. six times the sum of x and y squared decreased by three times the sum of x and half of y squared

48. **CRITICAL THINKING** Tell whether the Commutative Property *always*, *sometimes*, or *never* holds for subtraction. Explain your reasoning.

49. WRITING IN MATH   Answer the question that was posed at the beginning of the lesson.

How can properties help you determine distances?

Include the following in your answer:
• an expression using the Commutative and Associative Properties that you could use to easily determine the distance from the airport to Five Points, and
• an explanation of how the Commutative and Associative Properties are useful in performing calculations.

Stop	Distance from Previous Stop
Five Points	0
Garnett	0.4
West End	1.5
Oakland City	1.5
Lakewood/ Ft. McPherson	1.1
East Point	1.9
College Park	1.8
Airport	0.8

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CONTENTS

50. Simplify $6(ac + 2b) + 2ac$.

   Ⓐ $10ab + 2ac$ Ⓑ $12ac + 20b$ Ⓒ $8ac + 12b$ Ⓓ $12abc + 2ac$

51. Which property can be used to show that the areas of the two rectangles are equal?

   Ⓐ Associative

   Ⓑ Commutative

   Ⓒ Distributive

   Ⓓ Reflexive

6 cm

5 cm

5 cm

6 cm
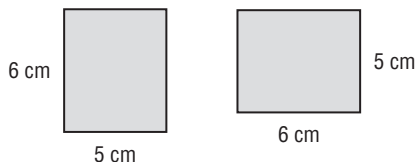
Maintain Your Skills

Mixed Review

Simplify each expression. *(Lesson 1-5)*

52. $5(2 + x) + 7x$ **53.** $3(5 + 2p)$ **54.** $3(a + 2b) - 3a$

55. $7m + 6(n + m)$ **56.** $(d + 5)f + 2f$ **57.** $t^2 + 2t^2 + 4t$

58. Name the property used in each step. *(Lesson 1-4)*

$$3(10 - 5 \cdot 2) + 21 \div 7 = 3(10 - 10) + 21 \div 7$$
$$= 3(0) + 21 \div 7$$
$$= 0 + 21 \div 7$$
$$= 0 + 3$$
$$= 3$$

Evaluate each expression. *(Lesson 1-2)*

59. $12(5) - 6(4)$ **60.** $7(0.2 + 0.5) - 0.6$ **61.** $8[6^2 - 3(2 + 5)] \div 8 + 3$

Getting Ready for the Next Lesson

PREREQUISITE SKILL Evaluate each expression for the given value of the variable. *(To review evaluating expressions, see Lesson 1-2.)*

62. If $x = 4$, then $2x + 7 = \underline{\ ?\ }$.   **63.** If $x = 8$, then $6x + 12 = \underline{\ ?\ }$.

64. If $n = 6$, then $5n - 14 = \underline{\ ?\ }$.   **65.** If $n = 7$, then $3n - 8 = \underline{\ ?\ }$.

66. If $a = 2$, and $b = 5$, then $4a + 3b = \underline{\ ?\ }$.

Practice Quiz 2   *Lessons 1-4 through 1-6*

Write the letters of the properties given in the right-hand column that match the examples in the left-hand column.

1. $28 + 0 = 28$ **a.** Distributive Property

2. $(18 - 7)6 = 11(6)$ **b.** Multiplicative Property of 0

3. $24 + 15 = 15 + 24$ **c.** Substitution Property of Equality

4. $8 \cdot 5 = 8 \cdot 5$ **d.** Multiplicative Identity Property

5. $(9 + 3) + 8 = 9 + (3 + 8)$ **e.** Multiplicative Inverse Property

6. $1(57) = 57$ **f.** Reflexive Property of Equality

7. $14 \cdot 0 = 0$ **g.** Associative Property

8. $3(13 + 10) = 3(13) + 3(10)$ **h.** Symmetric Property of Equality

9. If $12 + 4 = 16$, then $16 = 12 + 4$.   **i.** Commutative Property

10. $\frac{2}{5} \cdot \frac{5}{2} = 1$ **j.** Additive Identity Property

1-7 Logical Reasoning

What You'll Learn

- Identify the hypothesis and conclusion in a conditional statement.
- Use a counterexample to show that an assertion is false.

Vocabulary

- conditional statement
- if-then statement
- hypothesis
- conclusion
- deductive reasoning
- counterexample

How is logical reasoning helpful in cooking?

Popcorn is a popular snack with 16 billion quarts consumed in the United States each year. The directions at the right can help you make perfect popcorn. If the popcorn burns, then the heat was too high or the kernels heated unevenly.

Stovetop Popping

To pop popcorn on a stovetop, you need:
- A 3- to 4-quart pan with a loose lid that allows steam to escape
- Enough popcorn to cover the bottom of the pan, one kernel deep
- 1/4 cup of oil for every cup of kernels (Don't use butter!)

Heat the oil to 400–460 degrees Fahrenheit (if the oil smokes, it is too hot). Test the oil on a couple of kernels. When they pop, add the rest of the popcorn, cover the pan, and shake to spread the oil. When the popping begins to slow, remove the pan from the stovetop. The heated oil will pop the remaining kernels.

Source: Popcorn Board
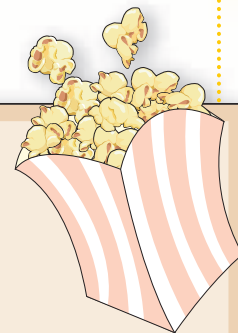
CONDITIONAL STATEMENTS   The statement *If the popcorn burns, then the heat was too high or the kernels heated unevenly* is called a conditional statement. **Conditional statements** can be written in the form *If A, then B*. Statements in this form are called **if-then statements**.

If   *A,*   then   *B.*

If   the popcorn burns,   then   the heat was too high or the kernels heated unevenly.

The part of the statement immediately following the word *if* is called the **hypothesis**.

The part of the statement immediately following the word *then* is called the **conclusion**.

Example 1 Identify Hypothesis and Conclusion

Identify the hypothesis and conclusion of each statement.

a. If it is Friday, then Madison and Miguel are going to the movies.

Recall that the hypothesis is the part of the conditional following the word *if* and the conclusion is the part of the conditional following the word *then*.

Hypothesis: it is Friday

Conclusion: Madison and Miguel are going to the movies

b. If $4x + 3 > 27$, then $x > 6$.

Hypothesis: $4x + 3 > 27$

Conclusion: $x > 6$

CONTENTS

Sometimes a conditional statement is written without using the words *if* and *then*. But a conditional statement can always be rewritten as an if-then statement. For example, the statement *When it is not raining, I ride my bike* can be written as *If it is not raining, then I ride my bike.*

Example 2 **Write a Conditional in If-Then Form**

Identify the hypothesis and conclusion of each statement. Then write each statement in if-then form.

a. I will go to the ball game with you on Saturday.

Hypothesis: it is Saturday

Conclusion: I will go to the ball game with you

If it is Saturday, then I will go to the ball game with you.

b. For a number x such that $6x - 8 = 16$, $x = 4$.

Hypothesis: $6x - 8 = 16$

Conclusion: $x = 4$

If $6x - 8 = 16$, then $x = 4$.

DEDUCTIVE REASONING AND COUNTEREXAMPLES **Deductive reasoning** is the process of using facts, rules, definitions, or properties to reach a valid conclusion. Suppose you have a true conditional and you know that the hypothesis is true for a given case. Deductive reasoning allows you to say that the conclusion is true for that case.

Example 3 **Deductive Reasoning**

Determine a valid conclusion that follows from the statement "If two numbers are odd, then their sum is even" for the given conditions. If a valid conclusion does not follow, write *no valid conclusion* and explain why.

a. The two numbers are 7 and 3.

7 and 3 are odd, so the hypothesis is true.

Conclusion: The sum of 7 and 3 is even.

CHECK $7 + 3 = 10$ $\checkmark$ The sum, 10, is even.

b. The sum of two numbers is 14.

The conclusion is true. If the numbers are 11 and 3, the hypothesis is true also. However, if the numbers are 8 and 6, the hypothesis is false. There is no way to determine the two numbers. Therefore, there is no valid conclusion.

Not all if-then statements are always true or always false. Consider the statement "If Luke is listening to CDs, then he is using his portable CD player." Luke may be using his portable CD player. However, he could also be using a computer, a car CD player, or a home CD player.

To show that a conditional is false, we can use a counterexample. A **counterexample** is a specific case in which a statement is false. It takes only one counterexample to show that a statement is false.

Example 4 Find Counterexamples

Find a counterexample for each conditional statement.

a. **If you are using the Internet, then you own a computer.**

   You could be using the Internet on a computer at a library.

b. **If the Commutative Property holds for multiplication, then it holds for division.**

   $2 \div 1 \overset{?}{=} 1 \div 2$

   $2 \neq 0.5$

Example 5 Find a Counterexample

Multiple-Choice Test Item

> Which numbers are counterexamples for the statement below?
>
> *If $x \div y = 1$, then x and y are whole numbers.*
>
> Ⓐ $x = 2, y = 2$ Ⓑ $x = 0.25, y = 0.25$
>
> Ⓒ $x = 1.2, y = 0.6$ Ⓓ $x = 6, y = 3$

Read the Test Item

Find the values of x and y that make the statement false.

Solve the Test Item

Replace x and y in the equation $x \div y = 1$ with the given values.

Ⓐ $x = 2, y = 2$

$2 \div 2 \overset{?}{=} 1$

$1 = 1$ ✓

The hypothesis is true and both values are whole numbers. The statement is true.

Ⓑ $x = 0.25, y = 0.25$

$0.25 \div 0.25 \overset{?}{=} 1$

$1 = 1$ ✓

The hypothesis is true, but 0.25 is not a whole number. Thus, the statement is false.

Ⓒ $x = 1.2, y = 0.6$

$1.2 \div 0.6 \overset{?}{=} 1$

$2 \neq 1$

The hypothesis is false, and the conclusion is false. However, this is not a counterexample. A counterexample is a case where the hypothesis is true and the conclusion is false.

Ⓓ $x = 6, y = 3$

$6 \div 3 \overset{?}{=} 1$

$2 \neq 1$

The hypothesis is false. Therefore, this is not a counterexample.

The only values that prove the statement false are $x = 0.25$ and $y = 0.25$. So these numbers are counterexamples. The answer is B.

Check for Understanding

Concept Check

1. **OPEN ENDED** Write a conditional statement and label its hypothesis and conclusion.

2. **Explain** why counterexamples are used.

3. **Explain** how deductive reasoning is used to show that a conditional is true or false.

Identify the hypothesis and conclusion of each statement.

4. If it is January, then it might snow.

5. If you play tennis, then you run fast.

6. If $34 - 3x = 16$, then $x = 6$.

Identify the hypothesis and conclusion of each statement. Then write the statement in if-then form.

7. Lance watches television when he does not have homework.

8. A number that is divisible by 10 is also divisible by 5.

9. A rectangle is a quadrilateral with four right angles.

Determine a valid conclusion that follows from the statement *If the last digit of a number is 2, then the number is divisible by 2* for the given conditions. If a valid conclusion does not follow, write *no valid conclusion* and explain why.

10. The number is 10,452.

11. The number is divisible by 2.

12. The number is 946.

Find a counterexample for each statement.

13. If Anna is in school, then she has a science class.

14. If you can read 8 pages in 30 minutes, then you can read any book in a day.

15. If a number x is squared, then $x^2 > x$.

16. If $3x + 7 \geq 52$, then $x > 15$.

17. Which number is a counterexample for the statement $x^2 > x$?

    Ⓐ 1   Ⓑ 4   Ⓒ 5   Ⓓ 8

Practice and Apply

Homework Help

For Exercises	See Examples
18–23	1
24–29	2
30–35	3
36–43	4

Extra Practice
See page 822.

Identify the hypothesis and conclusion of each statement.

18. If both parents have red hair, then their children have red hair.

19. If you are in Hawaii, then you are in the tropics.

20. If $2n - 7 > 25$, then $n > 16$.

21. If $4(b + 9) \leq 68$, then $b \leq 8$.

22. If $a = b$, then $b = a$.

23. If $a = b$ and $b = c$, then $a = c$.

Identify the hypothesis and conclusion of each statement. Then write the statement in if-then form.

24. The trash is picked up on Monday.

25. Greg will call after school.

26. A triangle with all sides congruent is an equilateral triangle.

27. The sum of the digits of a number is a multiple of 9 when the number is divisible by 9.

28. For $x = 8$, $x^2 - 3x = 40$.

29. $4s + 6 > 42$ when $s > 9$.

Determine whether a valid conclusion follows from the statement *If a VCR costs less than $150, then Ian will buy one* for the given condition. If a valid conclusion does not follow, write *no valid conclusion* and explain why.

30. A VCR costs $139.

31. A VCR costs $99.

32. Ian will not buy a VCR.

33. The price of a VCR is $199.

34. A DVD player costs $229.

35. Ian bought 2 VCRs.

Find a counterexample for each statement.

36. If you were born in Texas, then you live in Texas.

37. If you are a professional basketball player, then you play in the United States.

38. If a baby is wearing blue clothes, then the baby is a boy.

39. If a person is left-handed, then each member of that person's family is left-handed.

40. If the product of two numbers is even, then both numbers must be even.

41. If a whole number is greater than 7, then two times the number is greater than 16.

42. If $4n - 8 \geq 52$, then $n > 15$.

43. If $x \cdot y = 1$, then x or y must equal 1.

GEOMETRY For Exercises 44 and 45, use the following information.
If points P, Q, and R lie on the same line, then Q is between P and R.


44. Copy the diagram. Label the points so that the conditional is true.

45. Copy the diagram. Provide a counterexample for the conditional.

46. RESEARCH On Groundhog Day (February 2) of each year, some people say that if a groundhog comes out of its hole and sees its shadow, then there will be six more weeks of winter weather. However, if it does not see its shadow, then there will be an early spring. Use the Internet or another resource to research the weather on Groundhog Day for your city for the past 10 years. Summarize your data as examples or counterexamples for this belief.

NUMBER THEORY For Exercises 47–49, use the following information.
Copy the Venn diagram and place the numbers 1 to 25 in the appropriate places on the diagram.

47. What conclusions can you make about the numbers and where they appear on the diagram?

48. What conclusions can you form about numbers that are divisible by 2 and 3?

49. Find a counterexample for your conclusions, if possible.
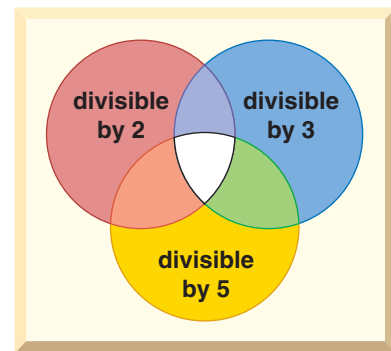
50. CRITICAL THINKING Determine whether the following statement is always true. If it is not, provide a counterexample.

*If the mathematical operation * is defined for all numbers a and b as a * b = a + 2b, then the operation * is commutative.*

51. WRITING IN MATH   Answer the question that was posed at the beginning of the lesson.

How is logical reasoning helpful in cooking?

Include the following in your answer:

- the hypothesis and conclusion of the statement *If you have small, underpopped kernels, then you have not used enough oil in your pan*, and
- examples of conditional statements used in cooking food other than popcorn.

52. GRID IN What value of n makes the following statement true?
If $14n - 12 \geq 100$, then $n \geq$ __?__ .

53. If # is defined as $\#x = \dfrac{x^3}{2}$, what is the value of #4?

   Ⓐ 8   Ⓑ 16   Ⓒ 32   Ⓓ 64

Maintain Your Skills

Mixed Review **Simplify each expression.** *(Lesson 1-6)*

54. $2x + 5y + 9x$ **55.** $a + 9b + 6b$ **56.** $\dfrac{3}{4}g + \dfrac{2}{5}f + \dfrac{5}{8}g$

57. $4(5mn + 6) + 3mn$ **58.** $2(3a + b) + 3b + 4$ **59.** $6x^2 + 5x + 3(2x^2) + 7x$

60. ENVIRONMENT According to the U.S. Environmental Protection Agency, a typical family of four uses 100 gallons of water flushing the toilet each day, 80 gallons of water showering and bathing, and 8 gallons of water using the bathroom sink. Write two expressions that represent the amount of water a typical family of four uses for these purposes in d days. *(Lesson 1-5)*

Name the property used in each expression. Then find the value of n. *(Lesson 1-4)*

61. $1(n) = 64$ **62.** $12 + 7 = n + 12$ **63.** $(9 - 7)5 = 2n$

64. $\dfrac{1}{4}n = 1$ **65.** $n + 18 = 18$ **66.** $36n = 0$

Solve each equation. *(Lesson 1-3)*

67. $5(7) + 6 = x$ **68.** $7(4^2) - 6^2 = m$ **69.** $p = \dfrac{22 - (13 - 5)}{28 \div 2^2}$

Write an algebraic expression for each verbal expression. *(Lesson 1-1)*

70. the product of 8 and a number x raised to the fourth power

71. three times a number n decreased by 10

72. twelve more than the quotient of a number a and 5

Getting Ready for the Next Lesson **PREREQUISITE SKILL** **Evaluate each expression. Round to the nearest tenth.**
*(To review **percents**, see pages 802 and 803.)*

73. 40% of 90   **74.** 23% of 2500   **75.** 18% of 950

76. 38% of 345   **77.** 42.7% of 528   **78.** 67.4% of 388

CONTENTS

1-8 Graphs and Functions

Vocabulary

- function
- coordinate system
- *x*-axis
- *y*-axis
- origin
- ordered pair
- *x*-coordinate
- *y*-coordinate
- independent variable
- dependent variable
- relation
- domain
- range

What You'll Learn

- Interpret graphs of functions.
- Draw graphs of functions.

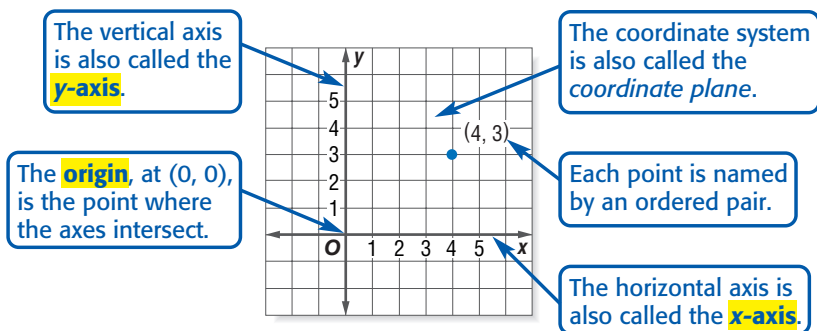**How** can real-world situations be modeled using graphs and functions?

Many athletes suffer concussions as a result of sports injuries. The graph shows the relationship between blood flow to the brain and the number of days after the concussion. The graph shows that as the number of days increases, the percent of blood flow increases.

Blood Flow After Concussion
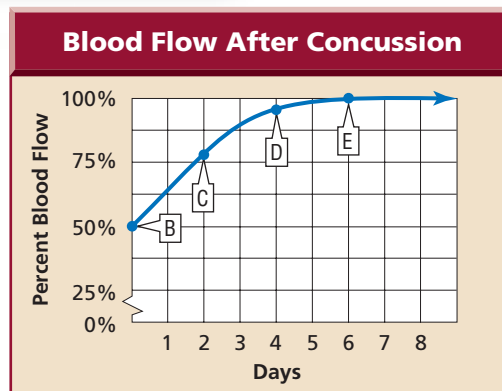
Source: *Scientific American*

INTERPRET GRAPHS The return of normal blood flow to the brain is said to be a function of the number of days since the concussion. A **function** is a relationship between input and output. In a function, the output depends on the input. There is exactly one output for each input.

A function is graphed using a **coordinate system**. It is formed by the intersection of two number lines, the *horizontal axis* and the *vertical axis*.

The vertical axis is also called the **y-axis**.

The **origin**, at (0, 0), is the point where the axes intersect.

The coordinate system is also called the *coordinate plane*.

Each point is named by an ordered pair.

The horizontal axis is also called the **x-axis**.

Each input *x* and its corresponding output *y* can be represented on a graph using ordered pairs. An **ordered pair** is a set of numbers, or *coordinates*, written in the form (*x*, *y*). The *x* value, called the **x-coordinate**, corresponds to the *x*-axis and the *y* value, or **y-coordinate**, corresponds to the *y*-axis.

Example 1 *Identify Coordinates*

SPORTS MEDICINE Refer to the application above. Name the ordered pair at point *C* and explain what it represents.

Point *C* is at 2 along the *x*-axis and about 80 along the *y*-axis. So, its ordered pair is (2, 80). This represents 80% normal blood flow 2 days after the injury.

CONTENTS

In Example 1, the percent of normal blood flow depends on the number of days from the injury. Therefore, the number of days from the injury is called the **independent variable** or *quantity*, and the percent of normal blood flow is called the **dependent variable** or *quantity*. Usually the independent variable is graphed on the horizontal axis and the dependent variable is graphed on the vertical axis.

Example 2 Independent and Dependent Variables

Identify the independent and dependent variables for each function.

a. **In general, the average price of gasoline slowly and steadily increases throughout the year.**

Time is the independent variable as it is unaffected by the price of gasoline, and the price is the dependent quantity as it is affected by time.

b. **The profit that a business makes generally increases as the price of their product increases.**

In this case, price is the independent quantity. Profit is the dependent quantity as it is affected by the price.

Functions can be graphed without using a scale on either axis to show the general shape of the graph that represents a function.

Example 3 Analyze Graphs

a. **The graph at the right represents the speed of a school bus traveling along its morning route. Describe what is happening in the graph.**

bus is accelerating

maintaining speed

bus is slowing down

bus is stopped

Speed (mph)

Time (min)

At the origin, the bus is stopped. It accelerates and maintains a constant speed. Then it begins to slow down, eventually stopping. After being stopped for a short time, the bus accelerates again. The starting and stopping process repeats continually.
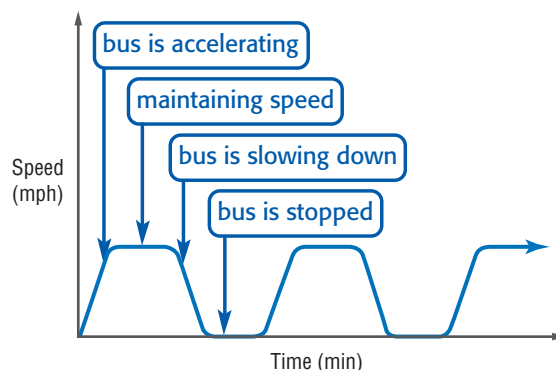
b. **Identify the graph that represents the altitude of a space shuttle above Earth, from the moment it is launched until the moment it lands.**

Graph A

Altitude

Time

Graph B

Altitude

Time

Graph C

Altitude

Time
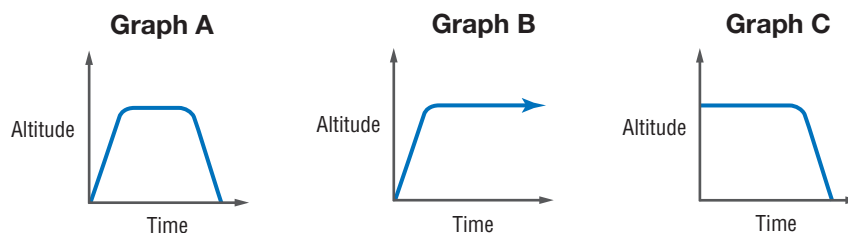
Before it takes off, the space shuttle is on the ground. It blasts off, gaining altitude until it reaches space where it orbits Earth at a constant height until it comes back to Earth. Graph A shows this situation.

CONTENTS

DRAW GRAPHS Graphs can be used to represent many real-world situations.

An electronics store is having a special sale. For every two DVDs you buy at the regular price of $29 each, you get a third DVD free.

a. **Make a table showing the cost of buying 1 to 5 DVDs.**

Number of DVDs	1	2	3	4	5
Total Cost ($)	29	58	58	87	116

b. **Write the data as a set of ordered pairs.**

The ordered pairs can be determined from the table. The number of DVDs is the independent variable, and the total cost is the dependent variable. So, the ordered pairs are (1, 29), (2, 58), (3, 58), (4, 87), and (5, 116).

c. **Draw a graph that shows the relationship between the number of DVDs and the total cost.**
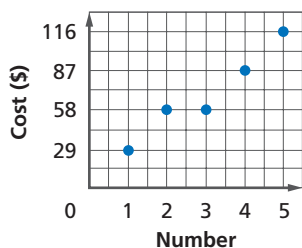
A set of ordered pairs, like those in Example 4, is called a **relation**. The set of the first numbers of the ordered pairs is the **domain**. The domain contains all values of the independent variable. The set of second numbers of the ordered pairs is the **range** of the relation. The range contains all values of the dependent variable.

Example 5 Domain and Range

JOBS **Rasha earns $6.75 per hour working up to 4 hours each day after school. Her weekly earnings are a function of the number of hours she works.**

a. **Identify a reasonable domain and range for this situation.**

The domain contains the number of hours Rasha works each week. Since she works up to 4 hours each weekday, she works up to 5×4 or 20 hours a week. Therefore, a reasonable domain would be values from 0 to 20 hours. The range contains her weekly earnings from $0 to $20 \times \$6.75$ or $135. Thus, a reasonable range is $0 to $135.

b. **Draw a graph that shows the relationship between the number of hours Rasha works and the amount she earns each week.**

Graph the ordered pairs (0, 0) and (20, 135). Since she can work any amount of time up to 20 hours, connect the two points with a line to include those points.
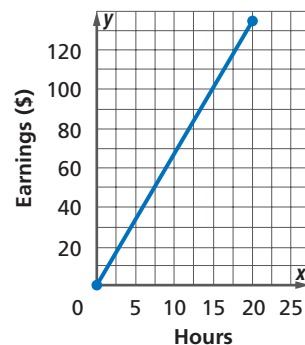
Study Tip

Different Representations
Example 4 illustrates several of the ways data can be represented—tables, ordered pairs, and graphs.

Check for Understanding

Concept Check
1. **Explain** why the order of the numbers in an ordered pair is important.
2. **Describe** the difference between dependent and independent variables.
3. **OPEN ENDED** Give an example of a relation. Identify the domain and range.

Guided Practice
4. The graph at the right represents Alexi's speed as he rides his bike. Give a description of what is happening in the graph.
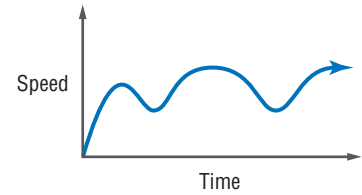
5. Identify the graph that represents the height of a skydiver just before she jumps from a plane until she lands.
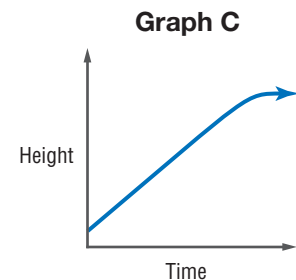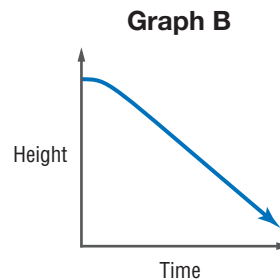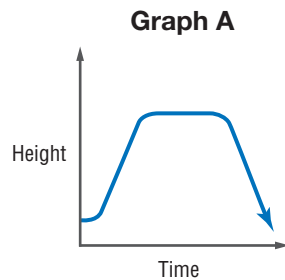
Applications
PHYSICAL SCIENCE For Exercises 6–8, use the table and the information.
During an experiment, the students of Ms. Roswell's class recorded the height of an object above the ground at several intervals after it was dropped from a height of 5 meters. Their results are in the table below.

Time (s)	0	0.2	0.4	0.6	0.8	1
Height (cm)	500	480	422	324	186	10

6. Identify the independent and dependent variables.

7. Write a set of ordered pairs representing the data in the table.

8. Draw a graph showing the relationship between the height of the falling object and time.

9. **BASEBALL** Paul is a pitcher for his school baseball team. Draw a reasonable graph that shows the height of the baseball from the ground from the time he releases the ball until the time the catcher catches the ball. Let the horizontal axis show the time and the vertical axis show the height of the ball.
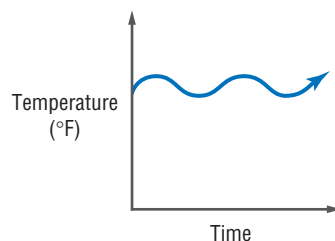
Practice and Apply

Homework Help

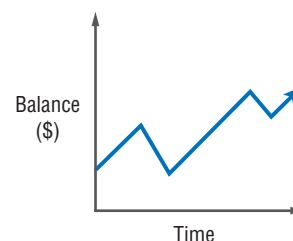| For Exercises	See Examples
10, 11	2
12, 13	3
14–21	4, 5

Extra Practice
See page 822.

10. The graph below represents Michelle's temperature when she was sick. Describe what is happening in the graph.


11. The graph below represents the balance in Rashaad's checking account. Describe what is happening in the graph.

12. TOYS Identify the graph that displays the speed of a radio-controlled car as it moves along and then hits a wall.

Graph A	Graph B	Graph C
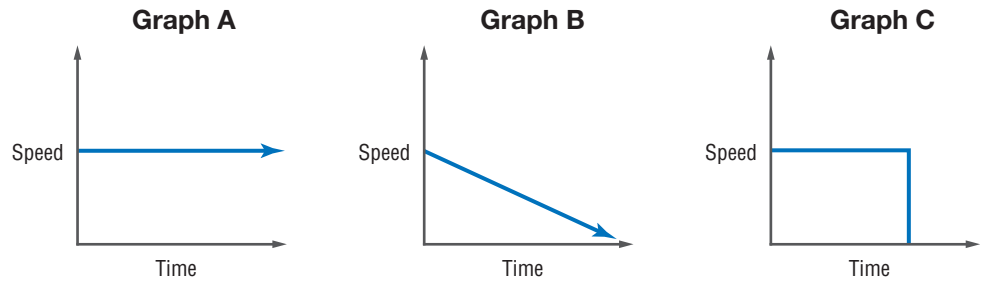
13. INCOME In general, as a person gets older, their income increases until they retire. Which of the graphs below represents this?

Graph A	Graph B	Graph C
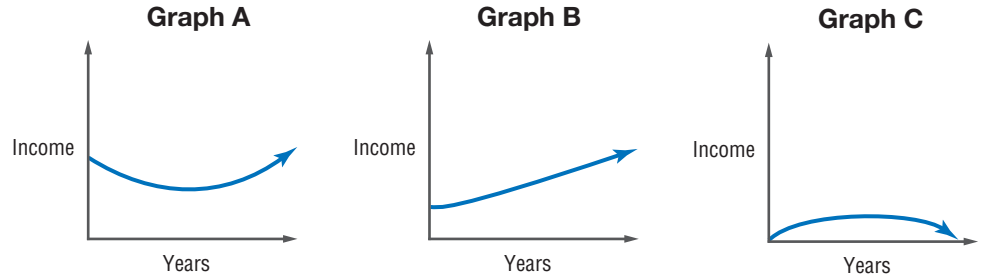
TRAVEL **For Exercises 14–16, use the table that shows the charges for parking a car in the hourly garage at an airport.**

Time Parked (h)	0–2	2–4	4–6	6–12	12–24
Cost ($)	1	2	4	5	30
After 24 hours: $15 per each 24-hour period					

14. Write the ordered pairs with whole-number coordinates that represent the cost of parking for up to 36 hours.

15. Draw a graph to show the cost of parking for up to 36 hours.

16. What is the cost of parking if you arrive on Monday at 7:00 A.M. and depart on Tuesday at 9:00 P.M.?

GEOMETRY **For Exercises 17–19, use the table that shows the relationships between the sum of the measures of the interior angles of convex polygons and the number of sides of the polygons.**

Polygon	triangle	quadrilateral	pentagon	hexagon	heptagon
Sides	3	4	5	6	7
Interior Angle Sum	180	360	540	720	900

17. Identify the independent and dependent variables.

18. Draw a graph of the data.

19. Use the data to predict the sum of the measures of the interior angles for an octagon, nonagon, and decagon.

20. CARS A car was purchased new in 1970. The owner has taken excellent care of the car, and it has relatively low mileage. Draw a reasonable graph to show the value of the car from the time it was purchased to the present.

21. CHEMISTRY When ice is exposed to temperatures above 32°F, it begins to melt. Draw a reasonable graph showing the relationship between the temperature of a block of ice as it is removed from a freezer and placed on a counter at room temperature. (*Hint*: The temperature of the water will not exceed the temperature of its surroundings.)

More About...

Cars

Most new cars lose 15 to 30 percent of their value in the first year. After about 12 years, more popular cars tend to increase in value.

Source: *Consumer Guide*

---

22. CRITICAL THINKING   Mallory is 23 years older than Lisa.

    a. Draw a graph showing Mallory's age as a function of Lisa's age for the first 40 years of Lisa's life.

    b. Find the point on the graph when Mallory is twice as old as Lisa.

23. WRITING IN MATH   Answer the question that was posed at the beginning of the lesson.

**How can real-world situations be modeled using graphs and functions?**

Include the following in your answer:
- an explanation of how the graph helps you analyze the situation,
- a summary of what happens during the first 24 hours from the time of a concussion, and
- an explanation of the time in which significant improvement occurs.

24. The graph shows the height of a model rocket shot straight up. How many seconds did it take for the rocket to reach its maximum height?

    Ⓐ 3   Ⓑ 4   Ⓒ 5   Ⓓ 6
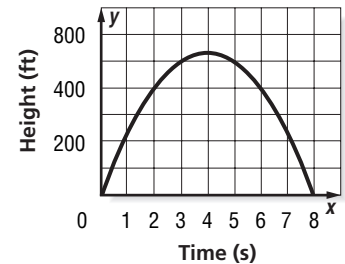
25. Andre owns a computer backup service. He charges his customers $2.50 for each backup CD. His expenses include $875 for the CD recording equipment and $0.35 for each blank CD. Which equation could Andre use to calculate his profit p for the recording of n CDs?

    Ⓐ $p = 2.15n - 875$ Ⓑ $p = 2.85 + 875$

    Ⓒ $p = 2.50 - 875.65$ Ⓓ $p = 875 - 2.15n$

Maintain Your Skills

Mixed Review   **Identify the hypothesis and conclusion of each statement.**   *(Lesson 1-7)*

26. You can send e-mail with a computer.

27. The express lane is for shoppers who have 9 or fewer items.

28. Name the property used in each step.   *(Lesson 1-6)*

$$ab(a + b) = (ab)a + (ab)b$$
$$= a(ab) + (ab)b$$
$$= (a \cdot a)b + a(b \cdot b)$$
$$= a^2b + ab^2$$

Name the property used in each statement. Then find the value of n.   *(Lesson 1-4)*

29. $(12 - 9)(4) = n(4)$ **30.** $7(n) = 0$ **31.** $n(87) = 87$

Getting Ready for the Next Lesson

32. PREREQUISITE SKILL   Use the information in the table to construct a bar graph.
*(To review **making bar graphs**, see pages 806 and 807.)*

U.S. Commercial Radio Stations by Format, 2000				
Format	country	adult contemporary	news/ talk	oldies	rock
Number	2249	1557	1426	1135	827

Source: *The World Almanac*

CONTENTS

Algebra Activity

Standards
3.02

Investigating Real-World Functions

The table shows the number of students enrolled in elementary and secondary schools in the United States for the given years.

Year	Enrollment (thousands)	Year	Enrollment (thousands)
1900	15,503	1970	45,550
1920	21,578	1980	41,651
1940	25,434	1990	40,543
1960	36,807	1998	46,327

Source: *The World Almanac*

Step 1 On grid paper, draw a vertical and horizontal axis as shown. Make your graph large enough to fill most of the sheet. Label the horizontal axis 0 to 120 and the vertical axis 0 to 60,000.

Step 2 To make graphing easier, let x represent the number of years since 1900. Write the eight ordered pairs using this method. The first will be (0, 15,503).

Step 3 Graph the ordered pairs on your grid paper.
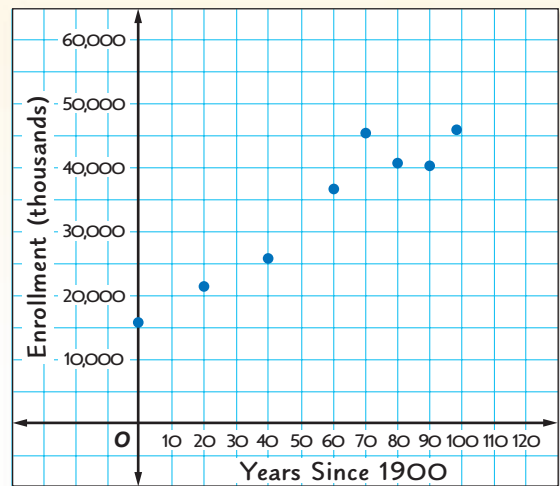
Analyze

1. Use your graph to estimate the number of students in elementary and secondary school in 1910 and in 1975.
2. Use your graph to estimate the number of students in elementary and secondary school in 2020.

Make a Conjecture

3. Describe the methods you used to make your estimates for Exercises 1 and 2.
4. Do you think your prediction for 2020 will be accurate? Explain your reasoning.
5. Graph this set of data, which shows the number of students per computer in U.S. schools. Predict the number of students per computer in 2010. Explain how you made your prediction.

Year	Students per Computer	Year	Students per Computer	Year	Students per Computer	Year	Students per Computer
1984	125	1988	32	1992	18	1996	10
1985	75	1989	25	1993	16	1997	7.8
1986	50	1990	22	1994	14	1998	6.1
1987	37	1991	20	1995	10.5	1999	5.7

Source: *The World Almanac*

CONTENTS

Statistics: Analyzing Data by Using Tables and Graphs

Vocabulary
- bar graph
- data
- circle graph
- line graph

What You'll Learn

- Analyze data given in tables and graphs (bar, line, and circle).
- Determine whether graphs are misleading.
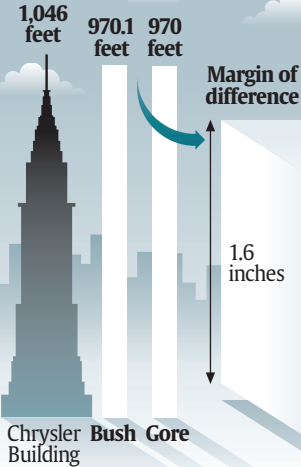
Why are graphs and tables used to display data?

For several weeks after Election Day in 2000, data regarding the presidential vote counts changed on a daily basis. The bar graph at the right illustrates just how close the election was at one point and the importance of each vote in the election. The graph allows you to compare the data visually.

USA TODAY Snapshots®

How close is Florida?

If a vote were a sheet of paper, a stack of George W. Bush's 2,910,299 votes in Florida would rise to a height of 970.1 feet. Al Gore's stack of 2,909,911 votes would rise to 970 feet.

1,046 feet — Chrysler Building
970.1 feet — Bush
970 feet — Gore
Margin of difference: 1.6 inches

Source: Martin Fertal
By Frank Pompa, USA TODAY

ANALYZE DATA A **bar graph** compares different categories of numerical information, or **data**, by showing each category as a bar whose length is related to the frequency. Bar graphs can also be used to display multiple sets of data in different categories at the same time. Graphs with multiple sets of data always have a key to denote which bars represent each set of data.

Example 1 Analyze a Bar Graph

The table shows the number of men and women participating in NCAA championship sports programs from 1995 to 1999.

NCAA Championship Sports Participation 1995–1999				
Year	'95–'96	'96–'97	'97–'98	'98–'99
Men	206,366	199,375	200,031	207,592
Women	125,268	129,295	133,376	145,832

Source: NCAA

This same data is displayed in a bar graph.

a. **Describe the general trend shown in the graph.**

The graph shows that the number of men has remained fairly constant while the number of women has been increasing.
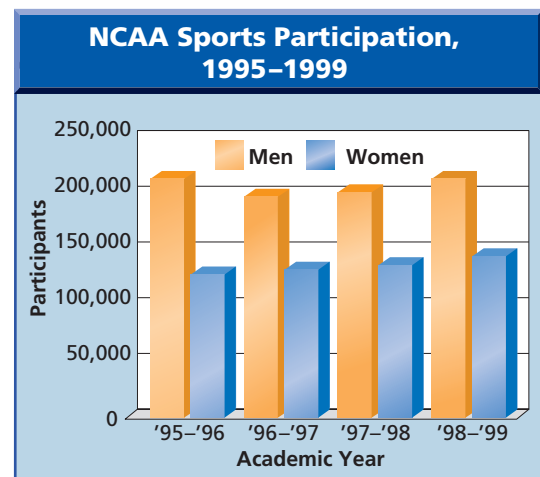
Study Tip

Graphs and Tables

Graphs are useful for visualizing data and for estimations. Tables are used when you need precise data for computation.

NCAA Sports Participation, 1995–1999

(Men, Women) bar graph with Participants axis from 0 to 250,000 and Academic Year axis '95–'96, '96–'97, '97–'98, '98–'99.

b. Approximately how many more men than women participated in sports during the 1997–1998 school year?

The bar for the number of men shows about 200,000 and the bar for the women shows about 130,000. So, there were approximately 200,000–130,000 or 70,000 more men than women participating in the 1997–1998 school year.

c. What was the total participation among men and women in the 1998–1999 academic year?

Since the table shows the exact numbers, use the data in it.

Number of men	plus	number of women	equals	total participation.
207,592	+	145,832	=	353,424

There was a total of 353,424 men and women participating in sports in the 1998–1999 academic year.

Another type of graph used to display data is a circle graph. A **circle graph** compares parts of a set of data as a percent of the whole set. The percents in a circle graph should always have a sum of 100%.

Example 2 Analyze a Circle Graph

A recent survey asked drivers in several cities across the United States if traffic in their area had gotten better, worse, or had not changed in the past five years. The results of the survey are displayed in the circle graph.

National Traffic Survey

3% Not Sure
26% Same
63% Worse
8% Better

Source: *USA TODAY*

a. If 4500 people were surveyed, how many felt that traffic had improved in their area?

The section of the graph representing people who said traffic is better is 8% of the circle, so find 8% of 4500.

| 8%	of	4500	equals	360.
0.08	×	4500	=	360

360 people said that traffic was better.

b. If a city with a population of 647,000 is representative of those surveyed, how many people could be expected to think that traffic conditions are worse?

63% of those surveyed said that traffic is worse, so find 63% of 647,000.

$0.63 \times 647{,}000 = 407{,}610$

Thus, 407,610 people in the city could be expected to say that traffic conditions are worse.
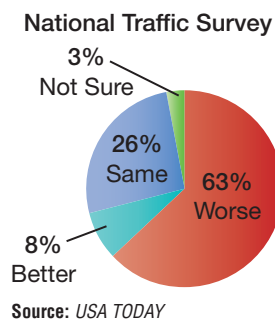
A third type of graph used to display data is a line graph. **Line graphs** are useful when showing how a set of data changes over time. They can also be helpful when making predictions.

Example 3 Analyze a Line Graph

EDUCATION Refer to the line graph below.

a. **Estimate the change in enrollment between 1995 and 1999.**

The enrollment for 1995 is about 14.25 million, and the enrollment for 1999 is about 14.9 million. So, the change in enrollment is $14.9 - 14.25$ or 0.65 million.

b. **If the rate of growth between 1998 and 1999 continues, predict the number of people who will be enrolled in higher education in the year 2005.**

Based on the graph, the increase in enrollment from 1998 to 1999 is 0.3 million. So, the enrollment should increase by 0.3 million per year.

$14.9 + 0.3(6) = 14.9 + 1.8$ Multiply the annual increase, 0.3, by the number of years, 6.

$= 16.7$ Enrollment in 2005 should be about 16.7 million.

Higher Education Enrollment, 1995–2000
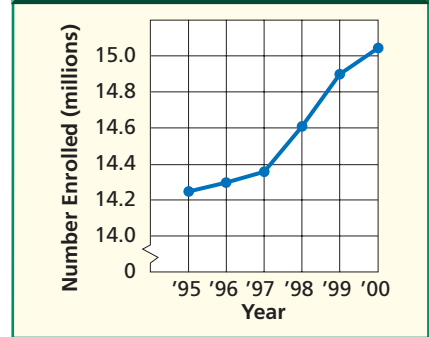
Source: U.S. National Center for Educational Statistics

Concept Summary			**Statistical Graphs**
Type of Graph	bar graph	circle graph	line graph
When to Use	to compare different categories of data	to show data as parts of a whole set of data	to show the change in data over time

---

MISLEADING GRAPHS   Graphs are very useful for displaying data. However, graphs that have been constructed incorrectly can be confusing and can lead to false assumptions. Many times these types of graphs are mislabeled, incorrect data is compared, or the graphs are constructed to make one set of data appear greater than another set. Here are some common ways that a graph may be misleading.

- Numbers are omitted on an axis, but no break is shown.
- The tick marks on an axis are not the same distance apart or do not have the same-sized intervals.
- The percents on a circle graph do not have a sum of 100.

Example 4 Misleading Graphs

AUTOMOBILES The graph shows the number of sport-utility vehicle (SUV) sales in the United States from 1990 to 1999. Explain how the graph misrepresents the data.

The vertical axis scale begins at 1 million. This causes the appearance of no vehicles sold in 1990 and 1991, and very few vehicles sold through 1994.
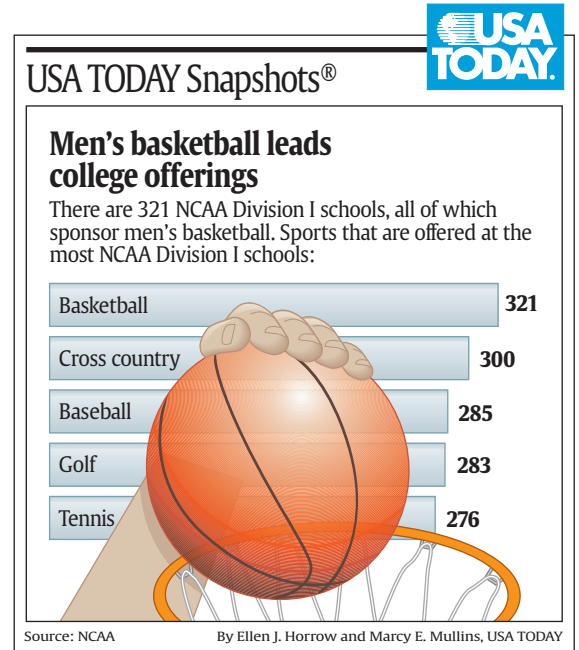
Sport-Utility Vehicle Sales, 1990–1999

Source: *The World Almanac*

CONTENTS
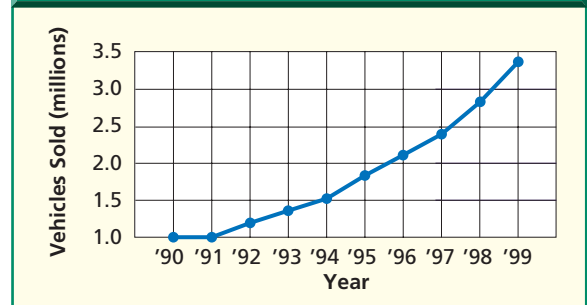
Check for Understanding

Concept Check

1. **Explain** the appropriate use of each type of graph.
   - circle graph
   - bar graph
   - line graph

2. **OPEN ENDED** Find a real-world example of a graph in a newspaper or magazine. Write a description of what the graph displays.

3. **Describe** ways in which a circle graph could be drawn so that it is misleading.

Guided Practice

SPORTS For Exercises 4 and 5, use the following information. There are 321 NCAA Division I schools. The graph at the right shows the sports that are offered at the most Division I schools.

4. How many more schools participate in basketball than in golf?

5. What sport is offered at the fewest schools?

USA TODAY Snapshots®

Men's basketball leads college offerings

There are 321 NCAA Division I schools, all of which sponsor men's basketball. Sports that are offered at the most NCAA Division I schools:

Sport	
Basketball	321
Cross country	300
Baseball	285
Golf	283
Tennis	276

Source: NCAA   By Ellen J. Horrow and Marcy E. Mullins, USA TODAY

EDUCATION For Exercises 6–9, use the table that shows the number of foreign students as a percent of the total college enrollment in the United States.

Country of Origin	Total Student Enrollment (%)
Australia	0.02
Canada	0.15
France	0.04
Germany	0.06
Italy	0.22
Spain	0.03
United Kingdom	0.05

Source: *Statistical Abstract of the United States*

6. There were about 14.9 million students enrolled in colleges in 1999. How many of these students were from Germany?

7. How many more students were from Canada than from the United Kingdom in 1999?

8. Would it be appropriate to display this data in a circle graph? Explain.

9. Would a bar or a line graph be more appropriate to display these data? Explain.

HOME ENTERTAINMENT For Exercises 10 and 11, refer to the graph.

10. Describe why the graph is misleading.

11. What should be done so that the graph displays the data more accurately?
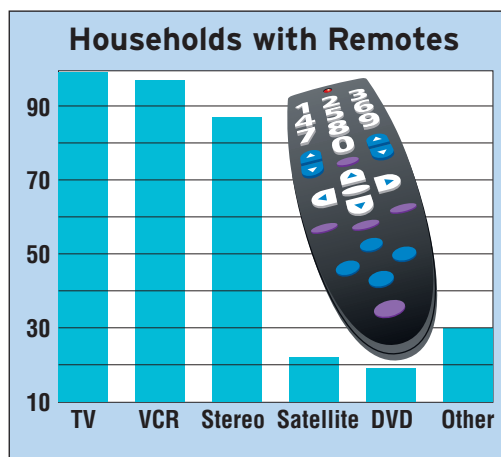

Households with Remotes

Practice and Apply

VIDEOGRAPHY For Exercises 12 and 13, use the table that shows the average cost of preparing one hour of 35-millimeter film versus one hour of digital video.

12. What is the total cost of using 35-millimeter film?

13. Estimate how many times as great the cost of using 35-millimeter film is as using digital video.

35 mm, editing video	
Film stock	$3110.40
Processing	621.00
Prep for telecine	60.00
Telecine	1000.00
Tape stock	73.20
Digital, editing on video	
Tape stock (original)	$10.00
Tape stock (back up)	10.00

When People Buy Books

21% Winter
19% Spring
15% Summer
44% Fall
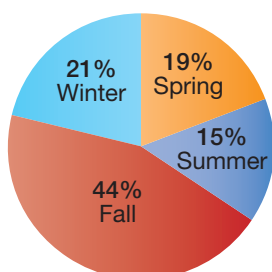
Source: *USA TODAY*

BOOKS For Exercises 14 and 15, use the graph that shows the time of year people prefer to buy books.

14. Suppose the total number of books purchased for the year was 25 million. Estimate the number of books purchased in the spring.

15. Suppose the manager of a bookstore has determined that she sells about 15,000 books a year. Approximately how many books should she expect to sell during the summer?

16. ENTERTAINMENT The line graph shows the number of cable television systems in the United States from 1995 to 2000. Explain how the graph misrepresents the data.

Cable Television Systems, 1995–2000

Systems (thousands)

Year
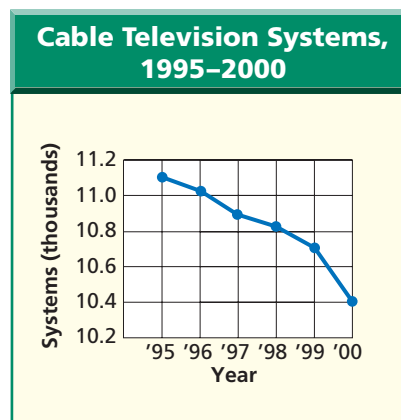
Data Source: *The World Almanac*

CONTENTS

17. FOOD Oatmeal can be found in 80% of the homes in the United States. The circle graph shows favorite oatmeal toppings. Is the graph misleading? If so, explain why and tell how the graph can be fixed so that it is not misleading.

Favorite Oatmeal Topping

9% Butter
38% Sugar
52% Milk
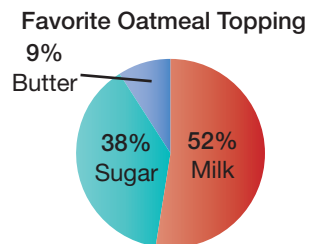
Data Source: NPD Group for Quaker Oats

18. CRITICAL THINKING The table shows the percent of United States households owning a color television for the years 1980 to 2000.

a. Display the data in a line graph that shows little increase in ownership.

b. Draw a line graph that shows a rapid increase in the number of households owning a color television.

c. Are either of your graphs misleading? Explain.

Households with Color Televisions	
Year	**Percent**
1980	83
1985	91
1990	98
1995	99
2000	99

Source: *The World Almanac*

19. WRITING IN MATH   Answer the question that was posed at the beginning of the lesson.

Why are graphs and tables used to display data?

Include the following in your answer:

• a description of how to use graphs to make predictions, and

• an explanation of how to analyze a graph to determine whether the graph is misleading.

NC Practice
Standardized Test Practice
Ⓐ Ⓑ Ⓒ Ⓓ

20. According to the graph, the greatest increase in temperature occurred between which two days?

Ⓐ 1 and 2   Ⓑ 6 and 7
Ⓒ 2 and 3   Ⓓ 5 and 6

21. A graph that is primarily used to show the change in data over time is called a

Ⓐ circle graph.   Ⓑ bar graph.
Ⓒ line graph.   Ⓓ data graph.

Average Temperatures
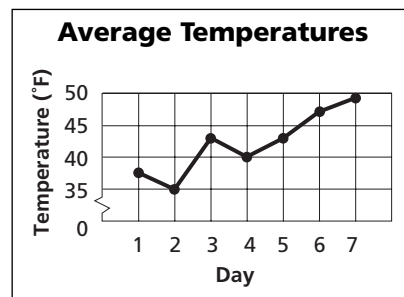
Maintain Your Skills

Mixed Review

22. PHYSICAL FITNESS Pedro likes to exercise regularly. On Mondays, he walks two miles, runs three miles, sprints one-half of a mile, and then walks for another mile. Sketch a graph that represents Mitchell's heart rate during his Monday workouts. *(Lesson 1-8)*

Find a counterexample for each statement. *(Lesson 1-7)*

23. If $x \leq 12$, then $4x - 5 \leq 42$.   **24.** If $x > 1$, then $x < \dfrac{1}{x}$.

25. If the perimeter of a rectangle is 16 inches, then each side is 4 inches long.

Simplify each expression. *(Lesson 1-6)*

26. $7a + 5b + 3b + 3a$ **27.** $4x^2 + 9x + 2x^2 + x$ **28.** $\dfrac{1}{2}n + \dfrac{2}{3}m + \dfrac{1}{2}m + \dfrac{1}{3}n$

www.algebra1.com/self_check_quiz/nc

CONTENTS

Standards
3.02

Statistical Graphs

You can use a computer spreadsheet program to display data in different ways. The data is entered into a table and then displayed in your chosen type of graph.

Example

**Use a spreadsheet to make a line graph of the data on sports equipment sales.**

In-line Skating and Wheel Sports Equipment Sales								
Year	1990	1992	1993	1994	1995	1996	1997	1998
Sales (million $)	150	268	377	545	646	590	562	515

Source: National Sporting Goods Association

Step 1 Enter the data in a spreadsheet. Use Column A for the years and Column B for the sales.

Step 2 Select the data to be included in your graph. Then use the graph tool to create the graph.

The spreadsheet will allow you to change the appearance of the graph by adding titles and axis labels, adjusting the scales on the axes, changing colors, and so on.
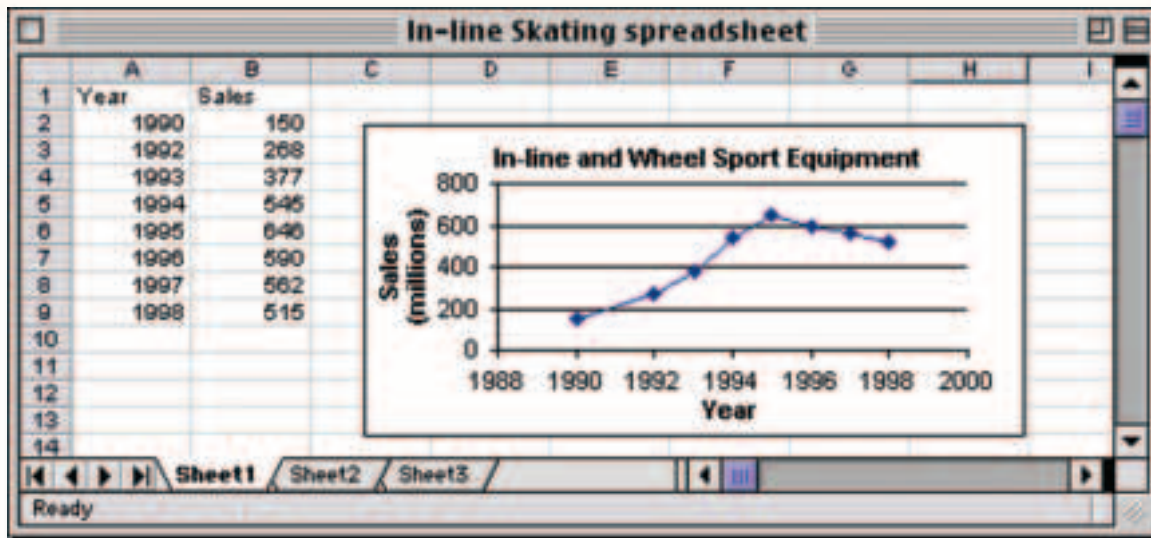
Exercises

For Exercises 1–3, use the data on snowmobile sales in the table below.

Snowmobile Sales								
Year	1990	1992	1993	1994	1995	1996	1997	1998
Sales (million $)	322	391	515	715	910	974	975	957

Source: National Sporting Goods Association

1. Use a spreadsheet program to create a line graph of the data.
2. Use a spreadsheet program to create a bar graph of the data.
3. Adjust the scales on each of the graphs that you created. Is it possible to create a misleading graph using a spreadsheet program? Explain.

CONTENTS

Vocabulary and Concept Check

additive identity (p. 21)
algebraic expression (p. 6)
Associative Property (p. 32)
bar graph (p. 50)
base (p. 7)
circle graph (p. 51)
Closure Property (p. 25)
coefficient (p. 29)
Commutative Property (p. 32)
conclusion (p. 37)
conditional statement (p. 37)
coordinate system (p. 43)
coordinates (p. 43)
counterexample (p. 38)
data (p. 50)
deductive reasoning (p. 38)
dependent quantity (p. 44)
dependent variable (p. 44)
Distributive Property (p. 26)
domain (p. 45)
element (p. 16)
equation (p. 16)

equivalent expressions (p. 29)
exponent (p. 7)
factors (p. 6)
function (p. 43)
horizontal axis (p. 43)
hypothesis (p. 37)
if-then statement (p. 37)
independent quantity (p. 44)
independent variable (p. 44)
inequality (p. 17)
like terms (p. 28)
line graph (p. 51)
multiplicative identity (p. 21)
Multiplicative Inverse Property (p. 22)
multiplicative inverses (p. 21)
Multiplicative Property of Zero (p. 21)
open sentence (p. 16)
order of operations (p. 11)
ordered pair (p. 43)
origin (p. 43)
power (p. 7)

product (p. 6)
range (p. 45)
reciprocal (p. 21)
Reflexive Property of Equality (p. 22)
relation (p. 45)
replacement set (p. 16)
set (p. 16)
simplest form (p. 29)
solution (p. 16)
solution set (p. 16)
solving an open sentence (p. 16)
Substitution Property of Equality (p. 22)
Symmetric Property of Equality (p. 22)
term (p. 28)
Transitive Property of Equality (p. 22)
variables (p. 6)
vertical axis (p. 43)
x-axis (p. 43)
x-coordinate (p. 43)
y-axis (p. 43)
y-coordinate (p. 43)

Choose the letter of the property that best matches each statement.

1. For any number a, $a + 0 = 0 + a = a$.
2. For any number a, $a \cdot 1 = 1 \cdot a = a$.
3. For any number, a, $a \cdot 0 = 0 \cdot a = 0$.
4. For any nonzero number a, there is exactly one number $\frac{1}{a}$ such that $\frac{1}{a} \cdot a = a \cdot \frac{1}{a} = 1$.
5. For any number a, $a = a$.
6. For any numbers a and b, if $a = b$, then $b = a$.
7. For any numbers a and b, if $a = b$, then a may be replaced by b in any expression.
8. For any numbers a, b, and c, if $a = b$ and $b = c$, then $a = c$.
9. For any numbers a, b, and c, $a(b + c) = ab + ac$.
10. For any numbers a, b, and c, $a + (b + c) = (a + b) + c$.

> a. Additive Identity Property
> b. Distributive Property
> c. Commutative Property
> d. Associative Property
> e. Multiplicative Identity Property
> f. Multiplicative Inverse Property
> g. Multiplicative Property of Zero
> h. Reflexive Property
> i. Substitution Property
> j. Symmetric Property
> k. Transitive Property

Lesson-by-Lesson Review

1-1 Variables and Expressions

See pages
6–9.

Concept Summary

- Variables are used to represent unspecified numbers or values.
- An algebraic expression contains letters and variables with an arithmetic operation.

Examples

1 Write an algebraic expression for *the sum of twice a number x and fifteen.*

$$\underbrace{\text{twice a number } x,}_{2x} \quad \underbrace{\text{sum of}}_{+} \quad \underbrace{\text{fifteen}}_{15}$$

The algebraic expression is $2x + 15$.

2 Write a verbal expression for $4x^2 - 13$.

Four times a number x squared minus thirteen.

Exercises Write an algebraic expression for each verbal expression.
See Examples 1 and 2 on pages 6 and 7.

11. a number x to the fifth power

12. five times a number x squared

13. the sum of a number x and twenty-one

14. the difference of twice a number x and 8

Evaluate each expression. *See Example 3 on page 7.*

15. 3^3

16. 2^5

17. 5^4

Write a verbal expression for each algebraic expression. *See Example 4 on page 7.*

18. $2p^2$

19. $3m^5$

20. $\frac{1}{2} + 2$

1-2 *Order of Operations*

See pages 11–15.

Concept Summary

- Expressions must be simplified using the order of operations.

  Step 1 Evaluate expressions inside grouping symbols.

  Step 2 Evaluate all powers.

  Step 3 Do all multiplications and/or divisions from left to right.

  Step 4 Do all additions and/or subtractions from left to right.

Example Evaluate $x^2 - (y + 2)$ if $x = 4$ and $y = 3$.

$$
\begin{aligned}
x^2 - (y + 2) &= 4^2 - (3 + 2) &&\text{Replace } x \text{ with 4 and } y \text{ with 3.}\\
&= 4^2 - 5 &&\text{Add 3 and 2.}\\
&= 16 - 5 &&\text{Evaluate power.}\\
&= 11 &&\text{Subtract 5 from 16.}
\end{aligned}
$$

Exercises Evaluate each expression. *See Examples 1–3 on pages 11 and 12.*

21. $3 + 2 \cdot 4$

22. $\frac{(10 - 6)}{8}$

23. $18 - 4^2 + 7$

24. $8(2 + 5) - 6$

25. $4(11 + 7) - 9 \cdot 8$

26. $288 \div [3(9 + 3)]$

27. $16 \div 2 \cdot 5 \cdot 3 \div 6$

28. $6(4^3 + 2^2)$

29. $(3 \cdot 1)^3 - \frac{(4 + 6)}{(5 \cdot 2)}$

Evaluate each expression if $x = 3$, $t = 4$, and $y = 2$. *See Example 4 on page 12.*

30. $t^2 + 3y$

31. xty^3

32. $\frac{ty}{x}$

33. $x + t^2 + y^2$

34. $3ty - x^2$

35. $8(x - y)^2 + 2t$

CONTENTS

1-3 Open Sentences

Concept Summary

- Open sentences are solved by replacing the variables in an equation with numerical values.
- Inequalities like $x + 2 \geq 7$ are solved the same way that equations are solved.

Example **Solve $5^2 - 3 = y$.**

$5^2 - 3 = y$ Original equation

$25 - 3 = y$ Evaluate the power.

$ 22 = y$ Subtract 3 from 25.

The solution is 22.

Exercises   **Solve each equation.**   *See Example 2 on page 17.*

36. $x = 22 - 13$

37. $y = 4 + 3^2$

38. $m = \dfrac{64 + 4}{17}$

39. $x = \dfrac{21 - 3}{12 - 3}$

40. $a = \dfrac{14 + 28}{4 + 3}$

41. $n = \dfrac{96 \div 6}{8 \div 2}$

42. $b = \dfrac{7(4 \cdot 3)}{18 \div 3}$

43. $\dfrac{6(7) - 2(3)}{4^2 - 6(2)}$

44. $y = 5[2(4) - 1^3]$

Find the solution set for each inequality if the replacement set is {4, 5, 6, 7, 8}.
See Example 3 on page 17.

45. $x + 2 > 7$

46. $10 - x < 7$

47. $2x + 5 \geq 15$

1-4 Identity and Equality Properties

Concept Summary

- Adding zero to a quantity or multiplying a quantity by one does not change the quantity.
- Using the Reflexive, Symmetric, Transitive, and Substitution Properties along with the order of operations helps in simplifying expressions.

Example **Evaluate $36 + 7 \cdot 1 + 5 (2 - 2)$. Name the property used in each step.**

$36 + 7 \cdot 1 + 5(2 - 2) = 36 + 7 \cdot 1 + 5(0)$ Substitution

$ = 36 + 7 + 5(0)$ Multiplicative Identity

$ = 36 + 7$ Multiplicative Prop. of Zero

$ = 43$ Substitution

Exercises   **Evaluate each expression. Name the property used in each step.**
See Example 2 on page 23.

48. $2[3 \div (19 - 4^2)]$

49. $\dfrac{1}{2} \cdot 2 + 2[2 \cdot 3 - 1]$

50. $4^2 - 2^2 - (4 - 2)$

51. $1.2 - 0.05 + 2^3$

52. $(7 - 2)(5) - 5^2$

53. $3(4 \div 4)^2 - \dfrac{1}{4}(8)$

CONTENTS

1-5 The Distributive Property

Concept Summary

- For any numbers a, b, and c, $a(b + c) = ab + ac$ and $(b + c)a = ba + ca$.
- For any numbers a, b, and c, $a(b - c) = ab - ac$ and $(b - c)a = ba - ca$.

Examples

1 Rewrite $5(t + 3)$ using the Distributive Property. Then simplify.

$$5(t + 3) = 5(t) + 5(3) \quad \text{Distributive Property}$$
$$= 5t + 15 \quad \text{Multiply.}$$

2 Simplify $2x^2 + 4x^2 + 7x$.

$$2x^2 + 4x^2 + 7x = (2 + 4)x^2 + 7x \quad \text{Distributive Property}$$
$$= 6x^2 + 7x \quad \text{Substitution}$$

Exercises Rewrite each product using the Distributive Property. Then simplify.
See Examples 1 and 2 on page 27.

54. $2(4 + 7)$ **55.** $8(15 - 6)$ **56.** $4(x + 1)$

57. $3\left(\dfrac{1}{3} - p\right)$ **58.** $6(a + b)$ **59.** $8(3x - 7y)$

Simplify each expression. If not possible, write *simplified*. *See Example 6 on page 29.*

60. $4a + 9a$ **61.** $4np + 7mp$ **62.** $3w - w + 4v - 3v$

63. $3m + 5m + 12n - 4n$ **64.** $2p(1 + 16r)$ **65.** $9y + 3y - 5x$

1-6 Commutative and Associative Properties

Concept Summary

- For any numbers a and b, $a + b = b + a$ and $a \cdot b = b \cdot a$.
- For any numbers a, b and c, $(a + b) + c = a + (b + c)$ and $(ab)c = a(bc)$.

Example Simplify $3x + 7xy + 9x$.

$$3x + 7xy + 9x = 3x + 9x + 7xy \quad \text{Commutative (+)}$$
$$= (3 + 9)x + 7xy \quad \text{Distributive Property}$$
$$= 12x + 7xy \quad \text{Substitution}$$

Exercises Simplify each expression. *See Example 3 on page 33.*

66. $3x + 4y + 2x$ **67.** $7w^2 + w + 2w^2$ **68.** $3\dfrac{1}{2}m + \dfrac{1}{2}m + n$

69. $6a + 5b + 2c + 8b$ **70.** $3(2 + 3x) + 21x$ **71.** $6(2n - 4) + 5n$

Write an algebraic expression for each verbal expression. Then simplify, indicating the properties used. *See Example 4 on page 34.*

72. five times the sum of x and y decreased by $2x$

73. twice the product of p and q increased by the product of p and q

74. six times a plus the sum of eight times b and twice a

75. three times the square of x plus the sum of x squared and seven times x

CONTENTS

1-7 Logical Reasoning

See pages 37–42.

Concept Summary

- Conditional statements can be written in the form *If A, then B*, where *A* is the hypothesis and *B* is the conclusion.
- One counterexample can be used to show that a statement is false.

Example **Identify the hypothesis and conclusion of the statement** *The trumpet player must audition to be in the band.* **Then write the statement in if-then form.**

Hypothesis: a person is a trumpet player

Conclusion: the person must audition to be in the band

If a person is a trumpet player, then the person must audition to be in the band.

Exercises **Identify the hypothesis and conclusion of each statement. Then, write each statement in if-then form.** *See Example 2 on page 38.*

76. School begins at 7:30 A.M. **77.** Triangles have three sides.

Find a counterexample for each statement. *See Example 4 on page 39.*

78. If $x > y$, then $2x > 3y$. **79.** If $a > b$ and $a > c$, then $b > c$.

1-8 Graphs and Functions

See pages 43–48.

Concept Summary

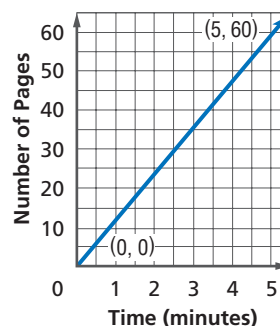- Graphs can be used to represent a function and to visualize data.

Example **A computer printer can print 12 pages of text per minute.**

a. Make a table showing the number of pages printed in 1 to 5 minutes.

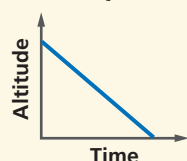| Time (min)	1	2	3	4	5
Pages	12	24	36	48	60

b. Sketch a graph that shows the relationship between time and the number of pages printed.
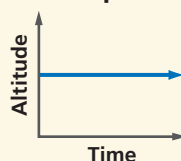

Exercises

80. Identify the graph that represents the altitude of an airplane taking off, flying for a while, then landing. *See Example 3 on page 44.*
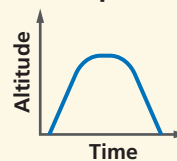

Graph A

Graph B

Graph C

CONTENTS

81. Sketch a reasonable graph that represents the amount of helium in a balloon if it is filled until it bursts. *See Examples 3–5 on pages 44 and 45.*

For Exercises 82 and 83, use the following information.
The planet Mars takes longer to orbit the sun than does Earth. One year on Earth is about 0.54 year on Mars. *See Examples 4 and 5 on page 45.*

82. Construct a table showing the relationship between years on Earth and years on Mars.

83. Draw a graph showing the relationship between Earth years and Mars years.

1-9 Statistics: Analyzing Data by Using Tables and Graphs

Concept Summary

• Bar graphs are used to compare different categories of data.
• Circle graphs are used to show data as parts of a whole set of data.
• Line graphs are used to show the change in data over time.

Example The bar graph shows ways people communicate with their friends.

a. About what percent of those surveyed chose e-mail as their favorite way to talk to friends?

The bar for e-mail is about halfway between 30% and 40%. Thus, about 35% favor e-mail.

b. What is the difference in the percent of people favoring letters and those favoring the telephone?

The bar for those favoring the telephone is at 60%, and the bar for letters is about 20%. So, the difference is 60 − 20 or 40%.

Favorite Method of Contacting Friends
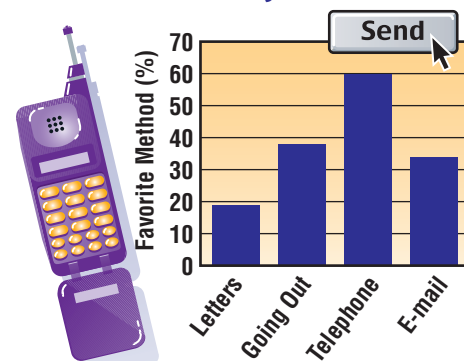

Source: *USA TODAY*

Exercises

CLASS TRIP **For Exercises 84 and 85, use the circle graph and the following information.**
A survey of the ninth grade class asked members to indicate their choice of locations for their class trip. The results of the survey are displayed in the circle graph. *See Example 2 on page 51.*

84. If 120 students were surveyed, how many chose the amusement park?

85. If 180 students were surveyed, how many more chose the amusement park than the water park?

9th Grade Class Survey

CONTENTS
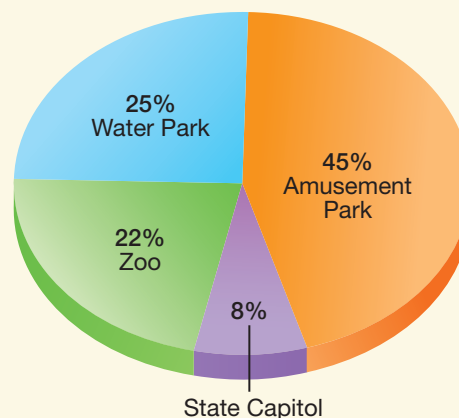
Vocabulary and Concepts

Choose the letter of the property that best matches each statement.

1. For any number a, $a = a$.
2. For any numbers a and b, if $a = b$, then b may be replaced by a in any expression or equation.
3. For any numbers a, b, and c, if $a = b$ and $b = c$, then $a = c$.

> **a.** Substitution Property of Equality
> **b.** Symmetric Property of Equality
> **c.** Transitive Property of Equality
> **d.** Reflexive Property of Equality

Skills and Applications

Write an algebraic expression for each verbal expression.

4. the sum of a number x and 13

5. the difference of 7 and a number x squared

Simplify each expression.

6. $5(9 + 3) - 3 \cdot 4$

7. $12 \cdot 6 \div 3 \cdot 2 \div 8$

Evaluate each expression if $a = 2$, $b = 5$, $c = 3$, and $d = 1$.

8. $a^2 b + c$

9. $(cd)^3$

10. $(a + d)c$

Solve each equation.

11. $y = (4.5 + 0.8) - 3.2$

12. $4^2 - 3(4 - 2) = x$

13. $\dfrac{2^3 - 1^3}{2 + 1} = n$

Evaluate each expression. Name the property used in each step.

14. $3^2 - 2 + (2 - 2)$

15. $(2 \cdot 2 - 3) + 2^2 + 3^2$

Rewrite each expression in simplest form.

16. $2m + 3m$

17. $4x + 2y - 2x + y$

18. $3(2a + b) - 5a + 4b$

Find a counterexample for each conditional statement.

19. If you run fifteen minutes today, then you will be able to run a marathon tomorrow.

20. If $2x - 3 < 9$, then $x \leq 6$.

Sketch a reasonable graph for each situation.

21. A basketball is shot from the free throw line and falls through the net.

22. A nickel is dropped on a stack of pennies and bounces off.

ICE CREAM **For Exercises 23 and 24, use the following information.**
A school survey at West High School determined the favorite flavors of ice cream are chocolate, vanilla, butter pecan, and bubble gum. The results of the survey are displayed in the circle graph.

23. If 200 students were surveyed, how many more chose chocolate than vanilla?

24. What was the total percent of students who chose either chocolate or vanilla?

Favorite Ice Cream

- 62% Chocolate
- 32% Vanilla
- 4% Bubble Gum
- 2% Butter Pecan
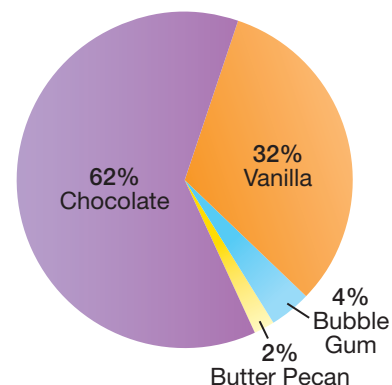
NC Practice

25. **STANDARDIZED TEST PRACTICE** Which number is a counterexample for the statement below?

If a is a prime number, then a is odd.

- Ⓐ 5
- Ⓑ 4
- Ⓒ 3
- Ⓓ 2

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**CONTENTS**

NC Practice

Part 1 Multiple Choice

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

1. The Maple Grove Warehouse measures 800 feet by 200 feet. If $\frac{3}{4}$ of the floor space is covered, how many square feet are *not* covered? **(Prerequisite Skill)**

   Ⓐ 4000   Ⓑ 40,000

   Ⓒ 120,000   Ⓓ 160,000

2. The radius of a circular flower garden is 4 meters. How many meters of edging will be needed to surround the garden? **(Prerequisite Skill)**

   (diagram of circle with 4 m radius)

   Ⓐ 7.14 m   Ⓑ 12.56 m
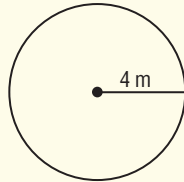
   Ⓒ 25.12 m   Ⓓ 20.24 m

3. The Johnson family spends about $80 per week on groceries. Approximately how much do they spend on groceries per year? **(Prerequisite Skill)**

   Ⓐ $400   Ⓑ $4000

   Ⓒ $8000   Ⓓ $40,000

4. Daria is making 12 party favors for her sister's birthday party. She has 50 stickers, and she wants to use as many of them as possible. If she puts the same number of stickers in each bag, how many stickers will she have left over? **(Prerequisite Skill)**

   Ⓐ 2   Ⓑ 4   Ⓒ 6   Ⓓ 8

The Princeton Review

Test-Taking Tip

Questions 1, 3, and 8 Read each question carefully. Be sure you understand what the question asks. Look for words like *not*, *estimate*, and *approximately*.

---

5. An auto repair shop charges $36 per hour, plus the cost of replaced parts. Which of the following expressions can be used to calculate the total cost of repairing a car, where h represents the number of hours of work and the cost of replaced parts is $85? **(Lesson 1-1)**

   Ⓐ $36 + h + 85$ Ⓑ $(85 \times h) + 36$

   Ⓒ $36 + 85 \times h$ Ⓓ $(36 \times h) + 85$

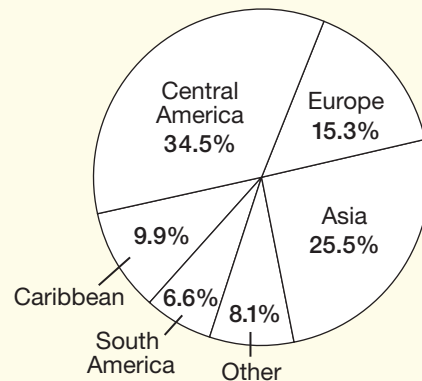6. Which expression is equivalent to $3(2x + 3) + 2(x + 1)$? **(Lessons 1-5 and 1-6)**

   Ⓐ $7x + 8$ Ⓑ $8x + 4$

   Ⓒ $8x + 9$ Ⓓ $8x + 11$

7. Find a counterexample for the following statement. **(Lesson 1-7)**
   If x is a positive integer, then x^2 is divisible by 2.

   Ⓐ 2   Ⓑ 3   Ⓒ 4   Ⓓ 6

8. The circle graph shows the regions of birth of foreign-born persons in the United States in 2000. According to the graph, which statement is *not* true? **(Lesson 1-9)**

   Regions of Birth

   (circle graph showing: Central America 34.5%, Europe 15.3%, Asia 25.5%, Other 8.1%, South America 6.6%, Caribbean 9.9%)

   Ⓐ More than $\frac{1}{3}$ of the foreign-born population is from Central America.

   Ⓑ More foreign-born people are from Asia than Central America.

   Ⓒ About half of the foreign-born population comes from Central America or Europe.

   Ⓓ About half of the foreign-born population comes from Central America, South America, or the Caribbean.

CONTENTS

Part 2 Short Response/Grid In

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

9. There are 32 students in the class. Five eighths of the students are girls. How many boys are in the class? (Prerequisite Skill)

10. Tonya bought two paperback books. One book cost $8.99 and the other $13.99. Sales tax on her purchase was 6%. How much change should she receive if she gives the clerk $25? (Prerequisite Skill)

11. According to the bar graph of the home runs hit by two baseball players, in which year was the difference between the numbers of home runs hit by the two players the least? (Prerequisite Skill)

Home Runs Hit Each Season
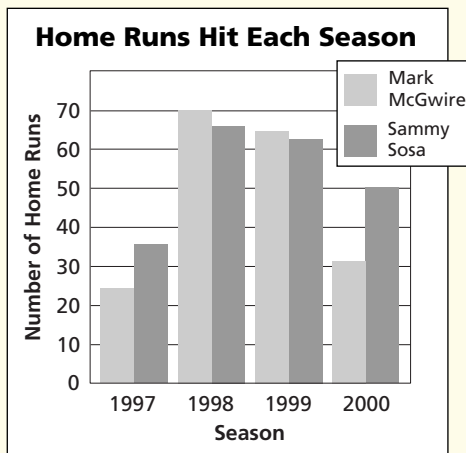
Part 3 Quantitative Comparison

Compare the quantity in Column A and the quantity in Column B. Then determine whether:

Ⓐ **the quantity in Column A is greater,**

Ⓑ **the quantity in Column B is greater,**

Ⓒ **the two quantities are equal, or**

Ⓓ **the relationship cannot be determined from the information given.**

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Column A	Column B
12. 15% of 80	25% of 50

(Prerequisite Skill)

Column A	Column B
13. $\dfrac{10}{\frac{2}{3}}$	1.5

(Prerequisite Skill)

Column A	Column B
14. $2x - 1$	$2x + 1$

(Lesson 1-3)

Column A	Column B
15. $\frac{1}{4}(a + b)c$	$\dfrac{ac + bc}{4}$

(Lesson 1-5)

Column A	Column B
16. $(26 \times 39) + (39 \times 13)$	$(39)^2$

(Lesson 1-5)

Part 4 Open Ended

Record your answers on a sheet of paper. Show your work.

17. Workers are draining water from a pond. They have an old pump and a new pump. The graphs below show how each pump drains water. (Lesson 1-8)
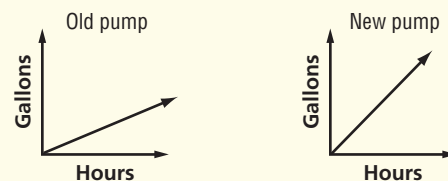
a. Describe how the old and new pumps are different in the amount of water they pump per hour.

b. Draw a graph that shows the gallons pumped per hour by both pumps at the same time.

c. Explain what the graph below tells about how the water is pumped out.