Data Analysis

Collecting and analyzing data allows you to make decisions and predictions about the future. In this unit, you will learn about statistics and probability.

Chapter 13
Statistics

Chapter 14
Probability

America Counts!

The U.S. government has been counting each person in the country since its first Census following independence was taken in 1790. Befitting the first Census of the 21st century, the Census Bureau allowed Census 2000 questionnaires to be completed electronically for the first time. In this project, you will see how data analysis can be used to compare statistics about a state of your choice to other states in the United States.

Log on to www.algebra1.com/webquest. Begin your WebQuest by reading the Task.

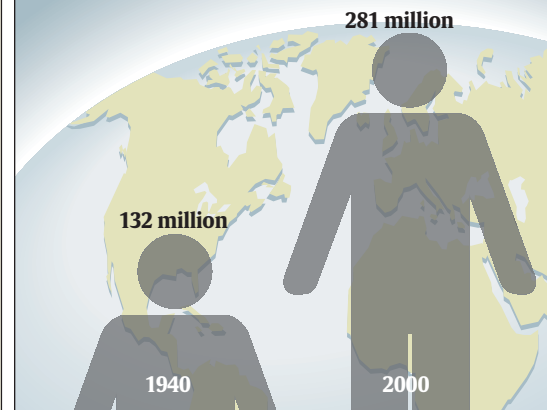Then continue working on your WebQuest as you study Unit 5.

Lesson	13-5	14-2
Page	742	766

USA TODAY Snapshots®

U.S. population growth
The U.S. population has more than doubled since 1940.

281 million

132 million

1940   2000

Source: U.S. Census Bureau   By Marcy E. Mullins, USA TODAY

CONTENTS

Chapter 13 Statistics

What You'll Learn

- **Lesson 13-1** Identify various sampling techniques.
- **Lesson 13-2** Solve problems by adding or subtracting matrices or by multiplying by a scalar.
- **Lesson 13-3** Interpret data displayed in histograms.
- **Lesson 13-4** Find the range, quartiles, and interquartile range of a set of data.
- **Lesson 13-5** Organize and use data in box-and-whisker plots.

Key Vocabulary

- **sample** (p. 708)
- **matrix** (p. 715)
- **histogram** (p. 722)
- **quartile** (p. 732)
- **box-and-whisker plot** (p. 737)

Why It's Important

Each day statistics are reported in the newspapers, in magazines, on television, and on the radio. These data involve business, government, ecology, sports, and many other topics. A basic knowledge of statistics allows you to interpret what you hear and read in the media. One important tool to help you understand the significance of a set of data is the box-and-whisker plot. *You will draw and use a box-and-whisker plot for data involving NASCAR racing in Lesson 13-5.*

Getting Started

▶ **Prerequisite Skills** To be successful in this chapter, you'll need to master these skills and be able to apply them in problem-solving situations. Review these skills before beginning Chapter 13.

For Lesson 13-1   **Use Logical Reasoning**

Find a counterexample for each statement. *(For review, see Lesson 1-7.)*

1. If $a + b = c$, then $a < c$.
2. If a flower is a rose, then it is red.
3. If Tara obeys the speed limit, then she will drive 45 miles per hour or less.
4. If a number is even, then it is divisible by 4.

For Lesson 13-4   **Find the Median**

Find the median for each set of data. *(For review, see pages 818 and 819.)*

5. 1, 7, 9, 15, 25, 59, 63
6. 0, 10, 2, 2, 9, 5, 4, 2, 8, 3, 8, 7, 3
7. 726, 411, 407, 407, 395, 355, 317, 235, 218, 211

For Lesson 13-5   **Graph Numbers on a Number Line**

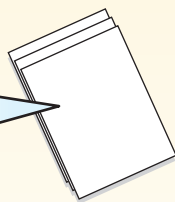Graph each set of numbers on a number line. *(For review, see Lesson 2-1.)*

8. {7, 9, 10, 13, 14}
9. {15, 17.5, 19, 20.5, 23}
10. {3.2, 4.8, 5.0, 5.7, 6.1}
11. {2.3, 2.8, 3.1, 3.7, 4.5}

FOLDABLES™
Study Organizer

Make this Foldable to help you organize information about statistics. Begin with three sheets of plain $8\frac{1}{2}$" by 11" paper.
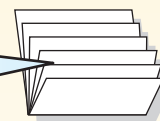
Step 1   **Stack Pages**
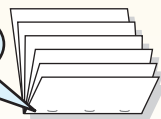
Stack sheets of paper with edges $\frac{3}{4}$ inch apart.

Step 2   **Fold Up Bottom Edges**
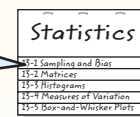
All tabs should be the same size.

Step 3   **Crease and Staple**

Staple along fold.

Step 4   **Turn and Label**

Label the tabs with topics from the chapter.

Statistics
13-1 Sampling and Bias
13-2 Matrices
13-3 Histograms
13-4 Measures of Variation
13-5 Box-and-Whisker Plots

Reading and Writing As you read and study the chapter, use each page to write notes and examples.

◀ **CONTENTS** ▶

13-1 Sampling and Bias

Vocabulary

- sample
- population
- census
- random sample
- simple random sample
- stratified random sample
- systematic random sample
- biased sample
- convenience sample
- voluntary response sample

Why is sampling important in manufacturing?

Manufacturing music CDs involves burning, or recording, copies from a master. However, not every burn is successful. It is costly and time-consuming to check every CD that is burned. Therefore, in order to monitor production, some CDs are picked at random and checked for defects.

SAMPLING TECHNIQUES When you wish to make an investigation, there are four ways that you can collect data.

- **published data** Use data that are already in a source like a newspaper or book.
- **observational study** Watch naturally occurring events and record the results.
- **experiment** Conduct an experiment and record the results.
- **survey** Ask questions of a group of people and record the results.

When performing an experiment or taking a survey, researchers often choose a sample. A **sample** is some portion of a larger group, called the **population**, selected to represent that group. If all of the units within a population are included, it is called a **census**. Sample data are often used to estimate a characteristic within an entire population, such as voting preferences prior to elections.

Population	Sample
all of the light bulbs manufactured on a production line	24 light bulbs selected from the production line
all of the water in a swimming pool	a test tube of water from the pool
all of the people in the United States	1509 people from throughout the United States

A **random sample** of a population is selected so that it is representative of the entire population. The sample is chosen without any preference. There are several ways to pick a random sample.

Key Concept — Random Samples

Type	Definition	Example
Simple Random Sample	A simple random sample is a sample that is as likely to be chosen as any other from the population.	The 26 students in a class are each assigned a different number from 1 to 26. Then three of the 26 numbers are picked at random.
Stratified Random Sample	In a stratified random sample, the population is first divided into similar, nonoverlapping groups. A simple random sample is then selected from each group.	The students in a school are divided into freshman, sophomores, juniors, and seniors. Then two students are randomly selected from each group of students.
Systematic Random Sample	In a systematic random sample, the items are selected according to a specified time or item interval.	Every 2 minutes, an item is pulled off the assembly line. or Every twentieth item is pulled off the assembly line.

CONTENTS

Example **1** *Classify a Random Sample*

ECOLOGY **Ten lakes are selected randomly from a list of all public-access lakes in Minnesota. Then 2 liters of water are drawn from 20 feet deep in each of the ten lakes.**

a. Identify the sample and suggest a population from which it was selected.

The sample is ten 2-liter containers of lake water, one from each of 10 lakes. The population is lake water from all of the public-access lakes in Minnesota.

b. Classify the sample as *simple,* *stratified,* **or** *systematic.*

This is a simple random sample. Each of the ten lakes was equally likely to have been chosen from the list.

BIASED SAMPLE   Random samples are unbiased. In a ==biased sample==, one or more parts of a population are favored over others.

Example **2** *Identify Sample as Biased or Unbiased*

Identify each sample as *biased* **or** *unbiased.* **Explain your reasoning.**

a. MANUFACTURING   Every 1000th bolt is pulled from the production line and measured for length.

The sample is chosen using a specified time interval. This is an unbiased sample because it is a systematic random sample.

b. MUSIC   Every tenth customer in line for a certain rock band's concert tickets is asked about his or her favorite rock band.

The sample is a biased sample because customers in line for concert tickets are more likely to name the band giving the concert as a favorite band.

Two popular forms of samples that are often biased include convenience samples and voluntary response samples.

Key Concept — *Biased Samples*

Type	Definition	Example
Convenience Sample	A convenience sample includes members of a population that are easily accessed.	To check spoilage, a produce worker selects 10 apples from the top of the bin. The 10 apples are unlikely to represent all of the apples in the bin.
Voluntary Response Sample	A voluntary response sample involves only those who want to participate in the sampling.	A radio call-in show records that 75% of its 40 callers voiced negative opinions about a local football team. Those 40 callers are unlikely to represent the entire local population. Volunteer callers are more likely to have strong opinions and are typically more negative than the entire population.

Example **3** *Identify and Classify a Biased Sample*

BUSINESS   The travel account records from 4 of the 20 departments in a corporation are to be reviewed. The accountant states that the first 4 departments to voluntarily submit their records will be reviewed.

a. Identify the sample and suggest a population from which it was selected.

The sample is the travel account records from 4 departments in the corporation. The population is the travel account records from all 20 departments in the corporation.

b. Classify the sample as *convenience* or *voluntary response*.

Since the departments voluntarily submit their records, this is a voluntary response sample.

Example 4 **Identify the Sample**

NEWS REPORTING For an article in the school paper, Rafael needs to determine whether students in his school believe that an arts center should be added to the school. He polls 15 of his friends who sing in the choir. Twelve of them think the school needs an arts center, so Rafael reports that 80% of the students surveyed support the project.

a. Identify the sample.

The sample is a group of students from the choir.

b. Suggest a population from which the sample was selected.

The population for the survey is all of the students in the school.

c. State whether the sample is *unbiased* (random) or *biased*. If unbiased, classify it as *simple*, *stratified*, or *systematic*. If biased, classify it as *convenience* or *voluntary response*.

The sample was not randomly selected from the entire student body. So the reported support is not likely to be representative of the student body. The sample is biased. Since Rafael polled only his friends, it is a convenience sample.

Check for Understanding

Concept Check

1. **Describe** how the following three types of sampling techniques are similar and how they are different.
   - simple random sample
   - stratified random sample
   - systematic random sample

2. **Explain** the difference between a convenience sample and a voluntary response sample.

3. **OPEN ENDED** Give an example of a biased sample.

Guided Practice

Identify each sample, suggest a population from which it was selected, and state whether it is *unbiased* (random) or *biased*. If unbiased, classify the sample as *simple*, *stratified*, or *systematic*. If biased, classify as *convenience* or *voluntary response*.

4. **NEWSPAPERS** The local newspaper asks readers to write letters stating their preferred candidate for mayor.

5. **SCHOOL** A teacher needs a sample of work from 4 students in her first-period math class to display at the school open house. She selects the work of the first 4 students who raise their hands.

6. **BUSINESS** A hardware store wants to assess the strength of nails it sells. Store personnel select 25 boxes at random from among all of the boxes on the shelves. From each of the 25 boxes, they select one nail at random and subject it to a strength test.

7. **SCHOOL** A class advisor hears complaints about an incorrect spelling of the school name on pencils sold at the school store. The advisor goes to the store and asks Namid to gather a sample of pencils and look for spelling errors. Namid grabs the closest box of pencils and counts out 12 pencils from the top of the box. She checks the pencils, returns them to the box, and reports the results to the advisor.

Practice and Apply

Identify each sample, suggest a population from which it was selected, and state whether it is *unbiased* (random) or *biased*. If unbiased, classify the sample as *simple*, *stratified*, or *systematic*. If biased, classify as *convenience* or *voluntary response*.

8. **SCHOOL**   Pieces of paper with the names of 3 sophomores are drawn from a hat containing identical pieces of paper with all sophomores' names.

9. **FOOD**   Twenty shoppers outside a fast-food restaurant are asked to name their preferred cola among two choices.

10. **RECYCLING**   An interviewer goes from house to house on weekdays between 9 A.M. and 4 P.M. to determine how many people recycle.

11. **POPULATION**   A state is first divided into its 86 counties and then 10 people from each county are chosen at random.

12. **SCOOTERS**   A scooter manufacturer is concerned about quality control. The manufacturer checks the first 5 scooters off the line in the morning and the last 5 off the line in the afternoon for defects.

13. **SCHOOL**   To determine who will speak for her class at the school board meeting, Ms. Finchie used the numbers appearing next to her students' names in her grade book. She writes each of the numbers on an identical piece of paper and shuffles the pieces of papers in a box. Without seeing the contents of the box, one student draws 3 pieces of paper from the box. The students whose numbers match the numbers chosen will speak for the class.

14. **FARMING**   An 8-ounce jar was filled with corn from a storage silo by dipping the jar into the pile of corn. The corn in the jar was then analyzed for moisture content.

15. **COURT**   The gender makeup of district court judges in the United States is to be estimated from a sample. All judges are grouped geographically by federal reserve districts. Within each of the 11 federal reserve districts, all judges' names are assigned a distinct random number. In each district, the numbers are then listed in order. A number between 1 and 20 inclusive is selected at random, and the judge with that number is selected. Then every 20th name after the first selected number is also included in the sample.

16. **TELEVISION**   A television station asks its viewers to share their opinions about a proposed golf course to be built just outside the city limits. Viewers can call one of two 900-numbers. One number represents a "yes" vote, and the other number represents a "no" vote.

17. **GOVERNMENT**   To discuss leadership issues shared by all United States Senators, the President asks 4 of his closest colleagues in the Senate to meet with him.

18. **FOOD**   To sample the quality of the Bing cherries throughout the produce department, the produce manager picks up a handful of cherries from the edge of one case and checks to see if these cherries are spoiled.

19. **MANUFACTURING**   During the manufacture of high-definition televisions, units are checked for defects. Within the first 10 minutes of a work shift, a television is randomly chosen from the line of completed sets. For the rest of the shift, every 15th television on the line is checked for defects.

More About...

Food

Michigan leads the nation in cherry production by growing about 219 million pounds of cherries per year.

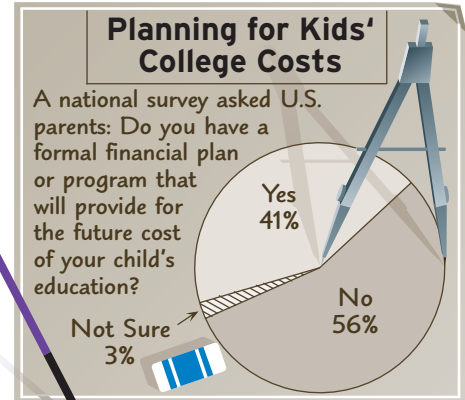**Source:** *World Book Encyclopedia*

Identify each sample, suggest a population from which it was selected, and state whether it is *unbiased* **(random) or** *biased*. **If unbiased, classify the sample as** *simple*, *stratified*, **or** *systematic*. **If biased, classify as** *convenience* **or** *voluntary response*.

20. BUSINESS To get reactions about a benefits package, a company uses a computer program to randomly pick one person from each of its departments.

21. MOVIES A magazine is trying to determine the most popular actor of the year. It asks its readers to mail the name of their favorite actor to the magazine's office.

COLLEGE **For Exercises 22 and 23, use the following information.**
The graph at the right reveals that 56% of survey respondents did not have a formal financial plan for a child's college tuition.

22. Write a statement to describe what you do know about the sample.

23. What additional information would you like to have about the sample to determine whether the sample is biased?

Planning for Kids' College Costs

A national survey asked U.S. parents: Do you have a formal financial plan or program that will provide for the future cost of your child's education?
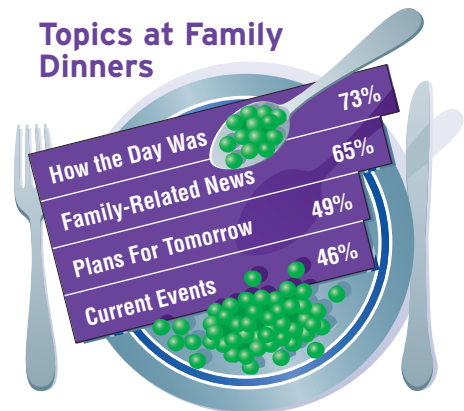
Yes 41%
No 56%
Not Sure 3%

Source: Yankelovich Partners for Microsoft Corp.

24. SCHOOL Suppose you want to sample the opinion of the students in your school about a new dress code. Describe an unbiased way to conduct your survey.

25. ELECTIONS Suppose you are running for mayor of your city and want to know if you are likely to be elected. Describe an unbiased way to poll the voters.

26. FAMILY Study the graph at the right. Describe the information that is revealed in the graph. What information is there about the type or size of the sample?

27. FARMING Suppose you are a farmer and want to know if your tomato crop is ready to harvest. Describe an unbiased way to determine whether the crop is ready to harvest.

Topics at Family Dinners

How the Day Was 73%
Family-Related News 65%
Plans For Tomorrow 49%
Current Events 46%

Source: National Pork Producers Council

28. MANUFACTURING Suppose you want to know whether the infant car seats manufactured by your company meet the government standards for safety. Describe an unbiased way to determine whether the seats meet the standards.

29. CRITICAL THINKING The following is a proposal for surveying a stratified random sample of the student body.

Divide the student body according to those who are on the basketball team, those who are in the band, and those who are in the drama club. Then take a simple random sample from each of the three groups. Conduct the survey using this sample.

Study the proposal. Describe its strengths and weaknesses. Is the sample a stratified random sample? Explain.

30. WRITING IN MATH   Answer the question that was posed at the beginning of the lesson.

Why is sampling important in manufacturing?

Include the following in your answer:
- an unbiased way to pick which CDs to check, and
- a biased way to pick which CDs to check.

31. To predict the candidate who will win the seat in city council, which method would give the newspaper the most accurate result?

Ⓐ Ask every 5th person that passes a reporter in the mall.
Ⓑ Use a list of registered voters and call every 20th person.
Ⓒ Publish a survey and ask readers to reply.
Ⓓ Ask reporters at the newspaper.

32. A cookie manufacturer plans to make a new type of cookie and wants to know if people will buy these cookies. For accurate results, which method should they use?

Ⓐ Ask visitors to their factory to evaluate the cookie.
Ⓑ Place a sample of the new cookie with their other cookies, and ask people to answer a questionnaire about the cookie.
Ⓒ Take samples to a school, and ask students to raise their hands if they like the cookie.
Ⓓ Divide the United States into 6 regions. Then pick 3 cities in each region at random, and conduct a taste test in each of the 18 cities.

Maintain Your Skills

Mixed Review   **Solve each equation.**   *(Lesson 12-9)*

33. $\dfrac{10}{3y} - \dfrac{5}{2y} = \dfrac{1}{4}$

34. $\dfrac{3}{r+4} - \dfrac{1}{r} = \dfrac{1}{r}$
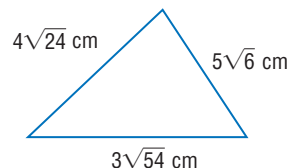
35. $\dfrac{1}{4m} + \dfrac{2m}{m-3} = 2$

Simplify.   *(Lesson 12-8)*

36. $\dfrac{2 + \dfrac{5}{x}}{\dfrac{x}{3} + \dfrac{5}{6}}$

37. $\dfrac{a + \dfrac{35}{a+12}}{a+7}$

38. $\dfrac{\dfrac{t^2-4}{t^2+5t+6}}{t-2}$

39. **GEOMETRY**   What is the perimeter of $\triangle ABC$?
(Lesson 11-2)

$4\sqrt{24}$ cm $5\sqrt{6}$ cm $3\sqrt{54}$ cm

Solve each equation by using the Quadratic Formula. Approximate any irrational roots to the nearest tenth.   *(Lesson 10-4)*

40. $x^2 - 6x - 40 = 0$

41. $6b^2 + 15 = -19b$

42. $2d^2 = 9d + 3$

Find each product.   *(Lesson 8-7)*

43. $(y + 5)(y + 7)$

44. $(c - 3)(c - 7)$

45. $(x + 4)(x - 8)$

Getting Ready for the Next Lesson   **BASIC SKILL**   **Find each sum or difference.**

46. $4.5 + 3.8$

47. $16.9 + 7.21$

48. $3.6 + 18.5$

49. $7.6 - 3.8$

50. $18 - 4.7$

**51.** $13.2 - 0.75$

CONTENTS

Reading Mathematics

Survey Questions

Even though taking a random sample eliminates bias or favoritism in the choice of a sample, questions may be worded to influence people's thoughts in a desired direction. Two different surveys on Internet sales tax had different results.
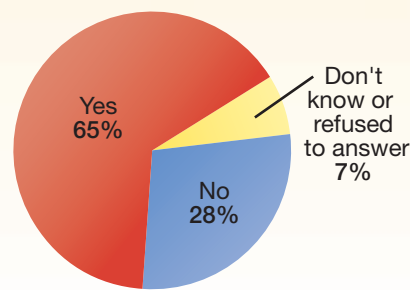
Question 1
Should there be sales tax on purchases made on the Internet?
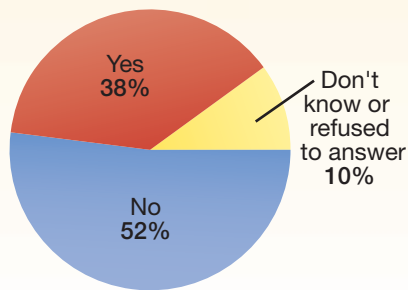
Question 2
Do you think people should or should not be required to pay the same sales tax for purchases made over the Internet that they would if they had bought the item in person at a local store?

Yes 38%
Don't know or refused to answer 10%
No 52%

Yes 65%
Don't know or refused to answer 7%
No 28%

Notice the difference in Questions 1 and 2. Question 2 includes more information. Pointing out that customers pay sales tax for items bought at a local store may give the people answering the survey a reason to answer "yes." Asking the question in that way probably led people to answer the way they did.

Because they are random samples, the results of both of these surveys are accurate. However, the results could be used in a misleading way by someone with an interest in the issue. For example, an Internet retailer would prefer to state the results of Question 1. Be sure to think about survey questions carefully to make sure that you interpret the results correctly.

Reading to Learn

For Exercises 1–2, tell whether each question is likely to bias the results. Explain your reasoning.

1. On a survey on environmental issues:
   a. "Due to diminishing resources, should a law be made to require recycling?"
   b. "Should the government require citizens to participate in recycling efforts?"
2. On a survey on education:
   a. "Should schools fund extracurricular sports programs?"
   b. "The budget of the River Valley School District is short of funds. Should taxes be raised in order for the district to fund extracurricular sports programs?"
3. Suppose you want to determine whether to serve hamburgers or pizza at the class party.
   a. Write a survey question that would likely produce biased results.
   b. Write a survey question that would likely produce unbiased results.

CONTENTS

13-2 Introduction to Matrices

Vocabulary

- matrix
- dimensions
- row
- column
- element
- scalar multiplication

What You'll Learn

- Organize data in matrices.
- Solve problems by adding or subtracting matrices or by multiplying by a scalar.

How are matrices used to organize data?

To determine the best type of aircraft to use for certain flights, the management of an airline company considers the following aircraft operating statistics.

Aircraft	Number of Seats	Airborne Speed (mph)	Possible Flight Distance (miles)	Fuel per Hour (gallons)	Operating Cost per Hour (dollars)
B747-100	462	512	2297	3517	7224
DC-10-10	297	496	1402	2311	5703
MD-11	259	527	3073	2464	6539
A300-600	228	475	1372	1505	4783

Source: Air Transport Association of America

The table has rows and columns of information. When we concentrate only on the numerical information, we see an array with 4 rows and 5 columns.

$$\begin{bmatrix} 462 & 512 & 2297 & 3517 & 7224 \\ 297 & 496 & 1402 & 2311 & 5703 \\ 259 & 527 & 3073 & 2464 & 6539 \\ 228 & 475 & 1372 & 1505 & 4783 \end{bmatrix}$$

This array of numbers is called a matrix.

Study Tip

Reading Math
A matrix is sometimes called an *ordered array*.

ORGANIZE DATA IN MATRICES   If you have ever used a spreadsheet program on the computer, you have worked with matrices. A **matrix** is a rectangular arrangement of numbers in rows and columns. A matrix is usually described by its **dimensions**, or the number of **rows** and **columns**, with the number of rows stated first. Each entry in a matrix is called an **element**.

Example 1 Name Dimensions of Matrices

State the dimensions of each matrix. Then identify the position of the circled element in each matrix.

a. [11 ⑮ 24]

This matrix has 1 row and 3 columns. Therefore, it is a 1-by-3 matrix.

The circled element is in the first row and the second column.

b. $\begin{bmatrix} -4 & 2 \\ 0 & 1 \\ ③ & -6 \end{bmatrix}$

This matrix has 3 rows and 2 columns. Therefore, it is a 3-by-2 matrix.

The circled element is in the third row and the first column.

Two matrices are *equal* only if they have the same dimensions and each element of one matrix is equal to the corresponding element in the other matrix.

$$\begin{bmatrix} 3 & 5 \\ -1 & 4 \end{bmatrix} = \begin{bmatrix} 3 & 5 \\ -1 & 4 \end{bmatrix} \qquad \begin{bmatrix} 2 & 4 \\ 1 & 7 \end{bmatrix} \neq \begin{bmatrix} 2 & 3 \\ 1 & 7 \end{bmatrix} \qquad \begin{bmatrix} 4 & 8 \\ 1 & -3 \end{bmatrix} \neq \begin{bmatrix} 4 & 8 & 0 \\ 1 & -3 & 0 \end{bmatrix}$$

MATRIX OPERATIONS If two matrices have the same dimensions, you can add or subtract them. To do this, add or subtract corresponding elements of the two matrices.

Example 2 Add Matrices

If $A = \begin{bmatrix} 3 & -4 & 7 \\ -1 & 6 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 7 & -4 & -2 \\ 1 & 6 & -3 \end{bmatrix}$, and $C = \begin{bmatrix} 3 & 6 \\ -4 & 5 \end{bmatrix}$, find each sum.

If the sum does not exist, write *impossible*.

a. $A + B$

$$A + B = \begin{bmatrix} 3 & -4 & 7 \\ -1 & 6 & 0 \end{bmatrix} + \begin{bmatrix} 7 & -4 & -2 \\ 1 & 6 & -3 \end{bmatrix} \qquad \text{Substitution}$$

$$= \begin{bmatrix} 3+7 & -4+(-4) & 7+(-2) \\ -1+1 & 6+6 & 0+(-3) \end{bmatrix} \qquad \text{Definition of matrix addition}$$

$$= \begin{bmatrix} 10 & -8 & 5 \\ 0 & 12 & -3 \end{bmatrix} \qquad \text{Simplify.}$$

b. $B + C$

$$B + C = \begin{bmatrix} 7 & -4 & -2 \\ 1 & 6 & -3 \end{bmatrix} + \begin{bmatrix} 3 & 6 \\ -4 & 5 \end{bmatrix} \qquad \text{Substitution}$$

Since B is a 2-by-3 matrix and C is a 2-by-2 matrix, the matrices do not have the same dimensions. Therefore, it is impossible to add these matrices.

Addition and subtraction of matrices can be used to solve real-world problems.

Example 3 Subtract Matrices

COLLEGE FOOTBALL The Division I-A college football teams with the five best records during the 1990s are listed below.

	Overall Record					Bowl Record		
	Wins	Losses	Ties			Wins	Losses	Ties
Florida State	109	13	1		Florida State	8	2	0
Nebraska	108	16	1		Nebraska	5	5	0
Marshall	114	25	0		Marshall	2	1	0
Florida	102	22	1		Florida	5	4	0
Tennessee	99	22	2		Tennessee	6	4	0

Use subtraction of matrices to determine the regular season records of these teams during the decade.

$$\begin{bmatrix} 109 & 13 & 1 \\ 108 & 16 & 1 \\ 114 & 25 & 0 \\ 102 & 22 & 1 \\ 99 & 22 & 2 \end{bmatrix} - \begin{bmatrix} 8 & 2 & 0 \\ 5 & 5 & 0 \\ 2 & 1 & 0 \\ 5 & 4 & 0 \\ 6 & 4 & 0 \end{bmatrix} = \begin{bmatrix} 109-8 & 13-2 & 1-0 \\ 108-5 & 16-5 & 1-0 \\ 114-2 & 25-1 & 0-0 \\ 102-5 & 22-4 & 1-0 \\ 99-6 & 22-4 & 2-0 \end{bmatrix}$$

$$= \begin{bmatrix} 101 & 11 & 1 \\ 103 & 11 & 1 \\ 112 & 24 & 0 \\ 97 & 18 & 1 \\ 93 & 18 & 2 \end{bmatrix}$$

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So, the regular season records of the teams can be described as follows.

Regular Season Record

	Wins	Losses	Ties
Florida State	101	11	1
Nebraska	103	11	1
Marshall	112	24	0
Florida	97	18	1
Tennessee	93	18	2

You can multiply any matrix by a constant called a *scalar*. This is called **scalar multiplication**. When scalar multiplication is performed, each element is multiplied by the scalar and a new matrix is formed.

Key Concept   **Scalar Multiplication of a Matrix**

$$m\begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} = \begin{bmatrix} ma & mb & mc \\ md & me & mf \end{bmatrix}$$

Example 4 Perform Scalar Multiplication

If $T = \begin{bmatrix} -4 & 2 \\ 0 & 1 \\ 3 & -6 \end{bmatrix}$, find $3T$.

$3T = 3\begin{bmatrix} -4 & 2 \\ 0 & 1 \\ 3 & -6 \end{bmatrix}$ Substitution

$= \begin{bmatrix} 3(-4) & 3(2) \\ 3(0) & 3(1) \\ 3(3) & 3(-6) \end{bmatrix}$ Definition of scalar multiplication

$= \begin{bmatrix} -12 & 6 \\ 0 & 3 \\ 9 & -18 \end{bmatrix}$ Simplify.

Check for Understanding

Concept Check   1. **Describe** the difference between a 2-by-4 matrix and a 4-by-2 matrix.

2. **OPEN ENDED**   Write two matrices whose sum is $\begin{bmatrix} 0 & 4 & 5 & -3 \\ 1 & -1 & 4 & 9 \end{bmatrix}$.

3. **FIND THE ERROR**   Hiroshi and Estrella are finding $-5\begin{bmatrix} -1 & 3 \\ -2 & 5 \end{bmatrix}$.

Hiroshi

$-5\begin{bmatrix} -1 & 3 \\ -2 & 5 \end{bmatrix} = \begin{bmatrix} 5 & 3 \\ 10 & 5 \end{bmatrix}$

Estrella

$-5\begin{bmatrix} -1 & 3 \\ -2 & 5 \end{bmatrix} = \begin{bmatrix} 5 & -15 \\ 10 & -25 \end{bmatrix}$

Who is correct? Explain your reasoning.

www.algebra1.com/extra_examples/nc

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Guided Practice State the dimensions of each matrix. Then, identify the position of the circled element in each matrix.

4. $\begin{bmatrix} 4 & 0 & 2 \\ 5 & -1 & -3 \\ 6 & 2 & 7 \end{bmatrix}$

5. $[3 \quad -3 \quad 1 \quad 9]$

6. $\begin{bmatrix} 5 \\ 2 \\ 1 \\ -3 \end{bmatrix}$

7. $\begin{bmatrix} 0.6 & 4.2 \\ -1.7 & 1.05 \\ 0.625 & -2.1 \end{bmatrix}$

If $A = \begin{bmatrix} 20 & -10 \\ 12 & 19 \end{bmatrix}$, $B = \begin{bmatrix} 15 & 14 \\ -10 & 6 \end{bmatrix}$, and $C = [-5 \quad 7]$, find each sum, difference, or product. If the sum or difference does not exist, write *impossible*.

8. $A + C$ 9. $B - A$ 10. $2A$ 11. $-4C$

Application **PIZZA SALES** For Exercises 12–16, use the following tables that list the number of pizzas sold at Sylvia's Pizza one weekend.

FRIDAY	Small	Medium	Large
Thin Crust	12	10	3
Thick Crust	11	8	8
Deep Dish	14	8	10

SATURDAY	Small	Medium	Large
Thin Crust	13	12	11
Thick Crust	1	5	10
Deep Dish	8	11	2

SUNDAY	Small	Medium	Large
Thin Crust	11	8	6
Thick Crust	1	8	11
Deep Dish	10	15	11

12. Create a matrix for each day's data. Name the matrices F, R, and N, respectively.

13. Does F equal R? Explain.

14. Create matrix T to represent $F + R + N$.

15. What does T represent?

16. Which type of pizza had the most sales during the entire weekend?

Practice and Apply

State the dimensions of each matrix. Then, identify the position of the circled element in each matrix.

17. $\begin{bmatrix} 2 & 1 \\ 5 & -8 \end{bmatrix}$

18. $\begin{bmatrix} -36 & 3 \\ 25 & -1 \\ 11 & 14 \end{bmatrix}$

19. $\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$

20. $\begin{bmatrix} -3 & 56 & -21 \\ 60 & 112 & -65 \end{bmatrix}$

21. $\begin{bmatrix} -4 & 0 & -2 \\ 5 & 1 & 12 \\ -6 & 3 & -7 \end{bmatrix}$

22. $\begin{bmatrix} 1 & -2 \\ 3 & 4 \\ 1 & 5 \\ -1 & 7 \end{bmatrix}$

23. $\begin{bmatrix} -5 & 3 & 1 \\ 4 & 0 & 2 \end{bmatrix}$

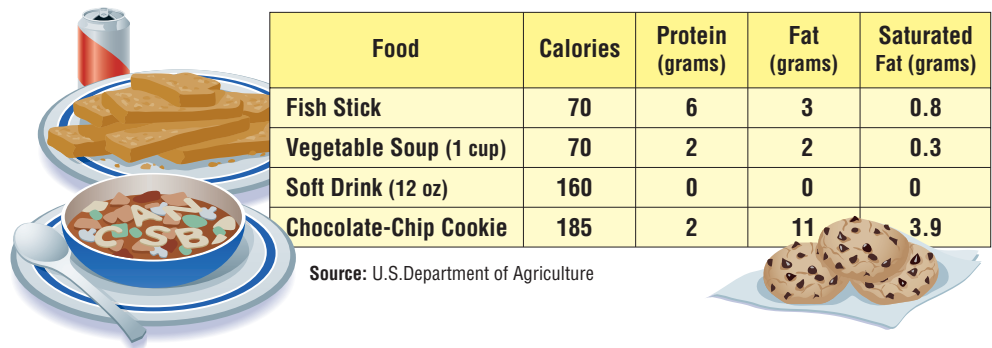24. $\begin{bmatrix} -6 & 3 \\ 5 & -4 \end{bmatrix}$

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25. Create a 2-by-3 matrix with 2 in the first row and first column and 5 in the second row and second column. The rest of the elements should be ones.

26. Create a 3-by-2 matrix with 8 in the second row and second column and 4 in the third row and second column. The rest of the elements should be zeros.

If $A = \begin{bmatrix} -1 & 5 & 9 \\ 0 & -4 & -2 \\ 3 & 7 & 6 \end{bmatrix}$, $B = \begin{bmatrix} -12 & 7 & -16 \\ 5 & 10 & 13 \\ 20 & 11 & 8 \end{bmatrix}$, $C = \begin{bmatrix} 34 & 91 & 63 \\ 81 & 79 & 60 \end{bmatrix}$, and

$D = \begin{bmatrix} -52 & 9 & 70 \\ -49 & -8 & 45 \end{bmatrix}$, find each sum, difference, or product. If the sum or difference does not exist, write *impossible.*

27. $A + B$ **28.** $C + D$ **29.** $C - D$ **30.** $B - A$

31. $5A$ **32.** $2C$ **33.** $A + C$ **34.** $B + D$

35. $2B + A$ **36.** $4A - B$ **37.** $2C - 3D$ **38.** $5D + 2C$

FOOD   For Exercises 39–41, use the table that shows the nutritional value of food.

Food	Calories	Protein (grams)	Fat (grams)	Saturated Fat (grams)
Fish Stick	70	6	3	0.8
Vegetable Soup (1 cup)	70	2	2	0.3
Soft Drink (12 oz)	160	0	0	0
Chocolate-Chip Cookie	185	2	11	3.9

Source: U.S.Department of Agriculture

39. If $F = [70 \quad 6 \quad 3 \quad 0.8]$ is a matrix representing the nutritional value of a fish stick, create matrices V, S, and C to represent vegetable soup, soft drink, and chocolate chip cookie, respectively.

40. Suppose Lakeisha has two fish sticks for lunch. Write a matrix representing the nutritional value of the fish sticks.

41. Suppose Lakeisha has two fish sticks, a cup of vegetable soup, a 12-ounce soft drink, and a chocolate chip cookie. Write a matrix representing the nutritional value of her lunch.

FUND-RAISING   For Exercises 42–44, use the table that shows the last year's sales of T-shirts for the student council fund-raiser.

Color	XS	S	M	L	XL
Red	18	28	32	24	21
White	24	30	45	47	25
Blue	17	19	26	30	28

42. Create a matrix to show the number of T-shirts sold last year according to size and color. Label this matrix N.

43. The student council anticipates a 20% increase in T-shirt sales this year. What value of the scalar r should be used so that rN results in a matrix that estimates the number of each size and color T-shirts needed this year?

44. Calculate rN, rounding appropriately, to show estimates for this year's sales.

FOOTBALL For Exercises 45–48, use the table that shows the passing performance of four National Football League quarterbacks.

1999 Regular Season

Quarterback	Attempts	Completions	Passing Yards	Touchdowns	Interceptions
Peyton Manning	533	331	4135	26	15
Rich Gannon	515	304	3840	24	14
Kurt Warner	499	325	4353	41	13
Steve Beuerlein	571	343	4436	36	15

2000 Regular Season

Quarterback	Attempts	Completions	Passing Yards	Touchdowns	Interceptions
Peyton Manning	571	357	4413	33	15
Rich Gannon	473	284	3430	28	11
Kurt Warner	347	235	3429	21	18
Steve Beuerlein	533	324	3730	19	18

Source: ESPN

45. Create matrix A for the 1999 data and matrix B for the 2000 data.

46. What are the dimensions of each matrix in Exercise 45?

47. Calculate $T = A + B$.

48. What does matrix T represent?

49. CRITICAL THINKING Suppose M and N are each 3-by-3 matrices. Determine whether each statement is *sometimes*, *always*, or *never* true.

  a. $M = N$

  b. $M + N = N + M$

  c. $M - N = N - M$

  d. $5M = M$

  e. $M + N = M$

  f. $5M = N$

50. WRITING IN MATH Answer the question that was posed at the beginning of the lesson.

How are matrices used to organize data?

Include the following in your answer:

- a comparison of a table and a matrix, and
- description of some real-world data that could be organized in a matrix.

51. Which of the following is equal to $\begin{bmatrix} 3 & 4 & 5 \\ -6 & -1 & 8 \end{bmatrix}$?

Ⓐ $\begin{bmatrix} -1 & 8 & 3 \\ -4 & 0 & 5 \end{bmatrix} + \begin{bmatrix} 4 & -4 & 2 \\ 2 & -1 & -2 \end{bmatrix}$

Ⓑ $\begin{bmatrix} 7 & -1 & 2 \\ 3 & 4 & -5 \end{bmatrix} + \begin{bmatrix} -4 & -3 & 3 \\ -3 & -5 & -3 \end{bmatrix}$

Ⓒ $\begin{bmatrix} 1 & -3 & 5 \\ 7 & -2 & 0 \end{bmatrix} + \begin{bmatrix} 2 & 7 & 0 \\ -13 & 1 & 8 \end{bmatrix}$

Ⓓ $\begin{bmatrix} 5 & 9 & -2 \\ 3 & 7 & 5 \end{bmatrix} + \begin{bmatrix} -2 & -5 & -3 \\ 3 & -8 & 3 \end{bmatrix}$

52. Suppose M and N are each 2-by-2 matrices. If $M + N = M$, which of the following is true?

Ⓐ $N = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

Ⓑ $N = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

Ⓒ $N = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Ⓓ $N = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

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MATRIX OPERATIONS You can use a graphing calculator to perform matrix operations. Use the EDIT command on the MATRX menu of a TI-83 Plus to enter each of the following matrices.

$$A = \begin{bmatrix} 7.9 & 5.4 & -6.8 \\ -5.9 & 4.4 & -7.7 \end{bmatrix}, B = \begin{bmatrix} -7.2 & -5.8 & 9.1 \\ 4.3 & -8.4 & 5.3 \end{bmatrix}, C = \begin{bmatrix} 9.8 & -1.2 & 5.2 \\ -7.8 & 5.1 & -9.0 \end{bmatrix}$$

Use these stored matrices to find each sum, difference, or product.

53. $A + B$ **54.** $C - B$ **55.** $B + C - A$ **56.** $1.8A$ **57.** $0.4C$

Maintain Your Skills

Mixed Review

PRINTING For Exercises 58 and 59, use the following information.
To determine the quality of calendars printed at a local shop, the last 10 calendars printed each day are examined. *(Lesson 13-1)*

58. Identify the sample.

59. State whether it is *unbiased* (random) or *biased*. If unbiased, classify the sample as *simple, stratified,* or *systematic*. If biased, classify as *convenience* or *voluntary response*.

Solve each equation. *(Lesson 12-9)*

60. $\dfrac{-4}{a+1} + \dfrac{3}{a} = 1$ **61.** $\dfrac{3}{x} + \dfrac{4x}{x-3} = 4$ **62.** $\dfrac{d+3}{d+5} + \dfrac{2}{d-9} = \dfrac{5}{2d+10}$

Find the nth term of each geometric sequence. *(Lesson 10-7)*

63. $a_1 = 4, n = 5, r = 3$ **64.** $a_1 = -2, n = 3, r = 7$ **65.** $a_1 = 4, n = 5, r = -2$

Factor each trinomial, if possible. If the trinomial cannot be factored using integers, write *prime*. *(Lesson 9-3)*

**66.** $b^2 + 7b + 12$ **67.** $a^2 + 2ab - 3b^2$ **68.** $d^2 + 8d - 15$

Getting Ready for the Next Lesson

PREREQUISITE SKILL For Exercises 69 and 70, use the graph that shows the amount of money in Megan's savings account. *(To review interpreting graphs, see Lesson 1-9.)*

69. Describe what is happening to Megan's bank balance. Give possible reasons why the graph rises and falls at particular points.

70. Describe the elements in the domain and range.

Megan's Savings Account
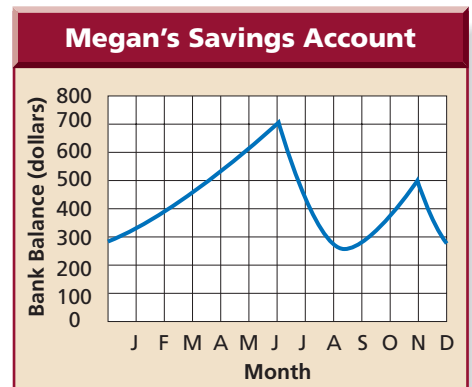
Practice Quiz 1

Lessons 13-1 and 13-2

Identify each sample, suggest a population from which it was selected, and state whether it is *unbiased* (random) or *biased*. If unbiased, classify the sample as *simple, stratified,* or *systematic*. If biased, classify as *convenience* or *voluntary response*. *(Lesson 13-1)*

1. Every other household in a neighborhood is surveyed to determine how to improve the neighborhood park.

2. Every other household in a neighborhood is surveyed to determine the favorite candidate for the state's governor.

Find each sum, difference, or product. *(Lesson 13-2)*

3. $\begin{bmatrix} -8 & 3 \\ -4 & -9 \end{bmatrix} + \begin{bmatrix} 5 & -7 \\ -1 & 0 \end{bmatrix}$ **4.** $\begin{bmatrix} -9 & 6 & 4 \\ -1 & 3 & 2 \end{bmatrix} - \begin{bmatrix} 7 & -2 & 8 \\ 5 & -3 & 1 \end{bmatrix}$ **5.** $3\begin{bmatrix} 8 & -3 & -4 & 5 \\ 6 & -1 & 2 & 10 \end{bmatrix}$

Histograms

- Interpret data displayed in histograms.
- Display data in histograms.

Vocabulary

- frequency table
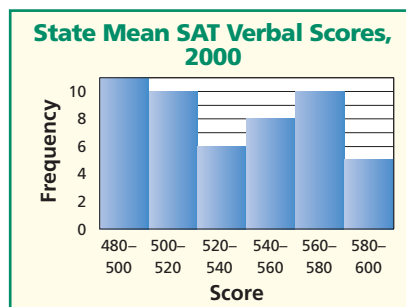- histogram
- measurement classes
- frequency

How are histograms used to display data?

A **frequency table** shows the frequency of events. The frequency table below shows the number of states with the mean SAT verbal and mathematics scores in each score interval. The data are from the 1999–2000 school year.
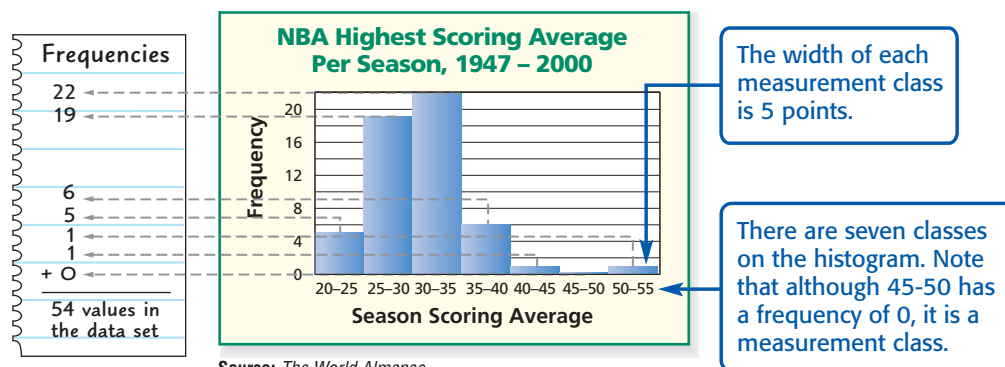
SAT Scores		
Score Interval	Verbal	Mathematics
	Number of States	Number of States
$480 \le s < 500$	11	5
$500 \le s < 520$	10	18
$520 \le s < 540$	6	5
$540 \le s < 560$	8	10
$560 \le s < 580$	10	5
$580 \le s < 600$	5	5
$600 \le s < 620$	0	2

Source: The College Board

The distribution of the mean scores on the SAT verbal exam is displayed in the graph.

State Mean SAT Verbal Scores, 2000

INTERPRET DATA IN HISTOGRAMS The graph above is called a histogram. A **histogram** is a bar graph in which the data are organized into equal intervals. In the histogram above, the horizontal axis shows the range of data values separated into **measurement classes**, and the vertical axis shows the number of values, or the **frequency**, in each class. Consider the histogram shown below.

NBA Highest Scoring Average Per Season, 1947 – 2000

Frequencies
22
19
6
5
1
1
+ O
54 values in the data set

The width of each measurement class is 5 points.

There are seven classes on the histogram. Note that although 45-50 has a frequency of 0, it is a measurement class.

Source: The World Almanac

A histogram is a visual summary of a frequency table.

Example 1 Determine Information from a Histogram

GEOGRAPHY Answer each question about the histogram shown below.

Population of Counties in Maine, 2000

a. In what measurement class does the median occur?

First, add the frequencies to determine the number of counties in Maine.

$7 + 4 + 3 + 1 + 0 + 1 = 16$

There are 16 counties, so the middle data value is between the 8th and 9th data values. Both the 8th and 9th data values are located in the 50–100 thousand measurement class. Therefore, the median occurs in the 50–100 thousand measurement class.

b. Describe the distribution of the data.

- Only two counties have populations above 150 thousand. It is likely that these counties contain the largest cities in Maine.
- There is a gap in the 200–250 thousand measurement class.
- Most of the counties have populations below 150 thousand.
- As population increases, the histogram shows that the number of counties decreases. We say that the distribution is *skewed*, or pulled in one direction away from the center. This distribution is *skewed to the left* because the majority of the data are located at the low end of the scale.

You can sometimes use the appearances of histograms to compare data.

NC Practice
Standardized Test Practice
Ⓐ Ⓑ Ⓒ Ⓓ

Example 2 Compare Data in Histograms

Multiple-Choice Test Item

Which group of students has a greater median height?

Heights of Classmates, Group A

Heights of Classmates, Group B

Ⓐ Group A

Ⓒ The medians are about the same.

Ⓑ Group B

Ⓓ cannot be determined

(continued on the next page)

Read the Test Item

You have two histograms depicting the heights of two groups of students. You are asked to determine which group of students has a greater median height.

Solve the Test Item

Study the histograms carefully. The measurement classes and the frequency scales are the same for each histogram. The distribution for Group A is somewhat *symmetrical* in shape, while the distribution for Group B is *skewed to the right*. This would indicate that Group B has the greater median height. To check this assumption, locate the measurement class of each median.

Group A

$4 + 6 + 8 + 5 + 4 + 1 = 28$
The median is between the 14th and 15th data values. The median is in the 140–150 measurement class.

Group B

$2 + 3 + 5 + 6 + 8 + 7 = 31$
The median is the 16th data value. The median is in the 150–160 measurement class.

This confirms that Group B has the greater median height. The answer is B.

DISPLAY DATA IN A HISTOGRAM   Data from a list or a frequency table can be used to create a histogram.

Example 3 Create a Histogram

SCHOOL   Create a histogram to represent the following scores for a 50-point mathematics test.

40, 34, 38, 23, 41, 39, 39, 34, 43, 44, 32, 44, 41, 39,
22, 47, 36, 25, 41, 30, 28, 37, 39, 33, 30, 40, 28

Step 1   **Identify the greatest and least values in the data set.**
The test scores range from 22 to 47 points.

Step 2   **Create measurement classes of equal width.**
For these data, use measurement classes from 20 to 50 with a 5-point interval for each class.

Step 3   **Create a frequency table using the measurement classes.**

Score Intervals	Tally	Frequency
$20 \leq s < 25$	II	2
$25 \leq s < 30$	III	3
$30 \leq s < 35$	卌 I	6
$35 \leq s < 40$	卌 II	7
$40 \leq s < 45$	卌 III	8
$45 \leq s < 50$	I	1

Step 4   **Draw the histogram.**
Use the measurement classes to determine the scale for the horizontal axis and the frequency values to determine the scale for the vertical axis. For each measurement class, draw a rectangle as wide as the measurement class and as tall as the frequency for the class. Label the axes and include a descriptive title for the histogram.

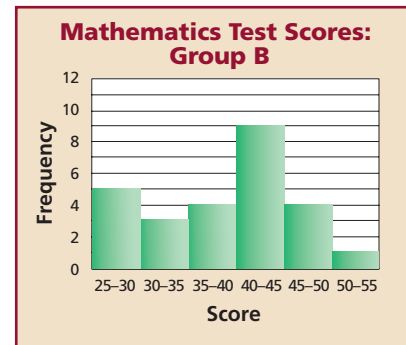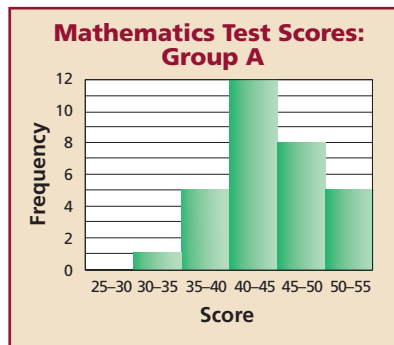**Mathematics Test Scores**

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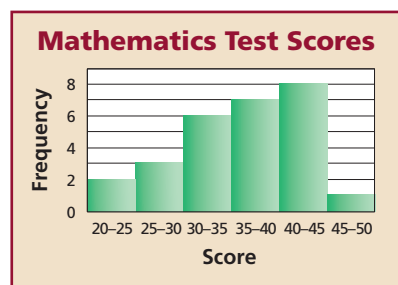
Concept Check
1. **Describe** how to create a histogram.
2. **Write** a compound inequality to represent all of the values v included in a 50–60 measurement class.
3. **OPEN ENDED** Write a set of data whose histogram would be skewed to the left.

Guided Practice
MONEY For Exercises 4 and 5, use the following histogram that shows the amount of money spent by several families during a holiday weekend.

Money Spent per Family During the Holiday Weekend

(Histogram: Frequency (y-axis, 0 to 10) vs Money Spent (dollars) (x-axis))

Money Spent (dollars)	Frequency
0–50	2
50–100	2
100–150	4
150–200	8
200–250	9
250–300	6
300–350	6
350–400	5
400–450	3
450–500	1
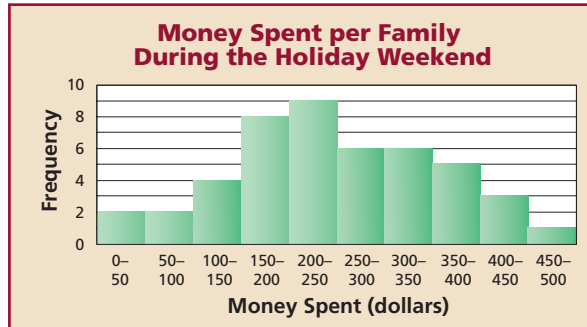
4. In what measurement class does the median occur?
5. Describe the distribution of the data.

SCHOOL For Exercises 6 and 7, use the following histograms.

Mathematics Test Scores: Group A

(Histogram: Frequency (y-axis, 0 to 12) vs Score (x-axis))

Score	Frequency
25–30	0
30–35	1
35–40	5
40–45	12
45–50	8
50–55	5

Mathematics Test Scores: Group B

(Histogram: Frequency (y-axis, 0 to 12) vs Score (x-axis))

Score	Frequency
25–30	5
30–35	3
35–40	4
40–45	9
45–50	4
50–55	1

6. Compare the medians of the two data sets.
7. Compare and describe the overall shape of each distribution of data.
8. **AIR TRAVEL** The busiest U.S. airports as determined by the number of passengers arriving and departing are listed below. Create a histogram.

Passenger Traffic at U.S. Airports, 2000			
Airport	Passengers (millions)	Airport	Passengers (millions)
Atlanta (Hartsfield)	80	Minneapolis/St. Paul	37
Chicago (O'Hare)	72	Phoenix (Sky Harbor)	36
Los Angeles	68	Detroit	36
Dallas/Fort Worth	61	Houston (George Bush)	35
San Francisco	41	Newark	34
Denver	39	Miami	34
Las Vegas (McCarran)	37	New York (JFK)	33

Source: Airports Council International

Online Research **Data Update** What are the current busiest airports? Visit www.algebra1.com/data_update to get statistics on airports.

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9. Which statement about the graph at the right is *not* correct?

   Ⓐ The data are skewed to the left.

   Ⓑ The median is in the 40–50 thousand measurement class.

   Ⓒ There are 32 employees represented by the graph.
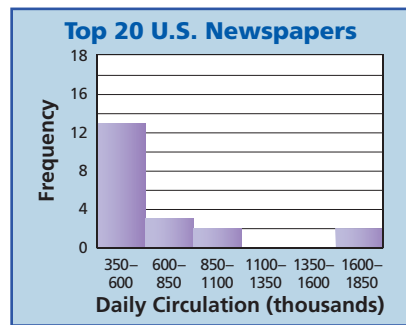
   Ⓓ The width of each measurement class is $10 thousand.
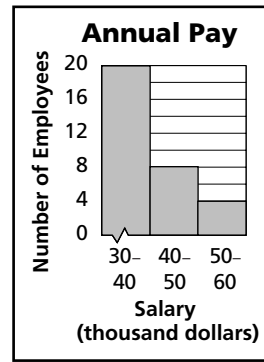
Practice and Apply

For each histogram, answer the following.

• In what measurement class does the median occur?

• Describe the distribution of the data.

10.

Source: *Editor & Publisher International Yearbook*

11.
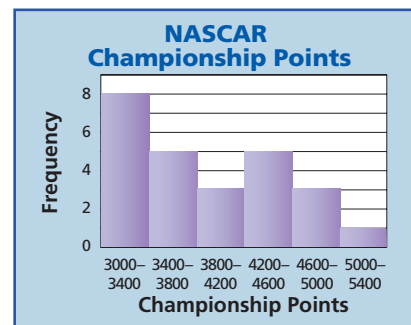
Source: *USA TODAY*

For each pair of histograms, answer the following.

• Compare the medians of the two data sets.

• Compare and describe the overall shape of each distribution of data.

12.
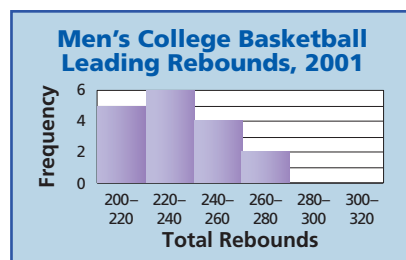
Source: *USA TODAY*
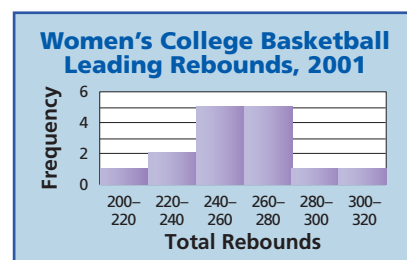
Source: *USA TODAY*

13.
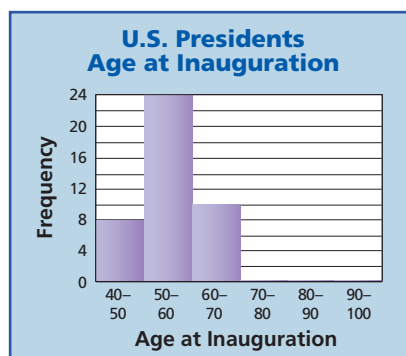
Source: *The World Almanac*
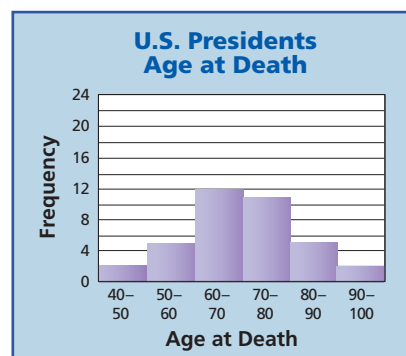
Source: *The World Almanac*

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Create a histogram to represent each data set.

14. Students' semester averages in a mathematics class: 96.53, 95.96, 94.25, 93.58, 91.91, 90.33, 90.27, 90.11, 89.30, 89.06, 88.33, 88.30, 87.43, 86.67, 86.31, 84.21, 83.53, 82.30, 78.71, 77.51, 73.83

15. Number of raisins found in a snack-size box: 54, 59, 55, 109, 97, 59, 102, 68, 104, 63, 101, 59, 59, 96, 58, 57, 63, 57, 94, 61, 104, 62, 58, 59, 102, 60, 54, 58, 53, 78

BASEBALL For Exercises 16 and 17, use the following table.

Payrolls for Major League Baseball Teams in 2000					
Team	**Payroll (millions)**	**Team**	**Payroll (millions)**	**Team**	**Payroll (millions)**
Yankees	$112	Orioles	$59	White Sox	$37
Braves	$94	Tigers	$59	Reds	$36
Red Sox	$91	Rockies	$56	Phillies	$36
Dodgers	$90	Padres	$55	Athletics	$32
Mets	$82	Blue Jays	$54	Pirates	$29
Indians	$77	Giants	$54	Expos	$28
Diamondbacks	$74	Angels	$53	Brewers	$26
Cardinals	$73	Devil Rays	$51	Marlins	$25
Rangers	$62	Astros	$51	Royals	$24
Mariners	$62	Cubs	$50	Twins	$15

Source: *USA TODAY*

16. Create a histogram to represent the payroll data.

17. On your histogram, locate and label the median team payroll.

ELECTIONS For Exercises 18–20, use the following table.

Percent of Eligible Voters Who Voted in the 2000 Presidential Election									
State	**Percent**	**State**	**Percent**	**State**	**Percent**	**State**	**Percent**	**State**	**Percent**
MN	68.75	WY	59.70	OH	55.76	MD	51.56	AR	47.79
ME	67.34	CT	58.40	ID	54.46	NJ	51.04	NM	47.40
AK	66.41	SD	58.24	RI	54.29	FL	50.65	SC	46.49
WI	66.07	MI	57.52	LA	54.24	NC	50.28	WV	45.74
VT	63.98	MO	57.49	KS	54.07	AL	49.99	CA	44.09
NH	62.33	WA	56.95	PA	53.66	IN	49.44	GA	43.84
MT	61.52	MA	56.92	IL	52.79	NY	49.42	NV	43.81
IA	60.71	CO	56.78	UT	52.61	TN	49.19	TX	43.15
OR	60.63	NE	56.44	VA	52.05	OK	48.76	AZ	42.26
ND	60.63	DE	56.22	KY	51.59	MS	48.57	HI	40.48

Source: *USA TODAY*

18. Determine the median of the data.

19. Create a histogram to represent the data.

20. Write a sentence or two describing the distribution of the data.

21. **RESEARCH** Choose your favorite professional sport. Use the Internet or other reference to find how many games each team in the appropriate league won last season. Use this information to create a histogram. Describe your histogram.

22. **CRITICAL THINKING** Create a histogram with a gap between 20 and 40, one item in the 50–55 measurement class, and the median value in the 50–55 measurement class.

www.algebra1.com/self_check_quiz/nc

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23. WRITING IN MATH   Answer the question that was posed at the beginning of the lesson.

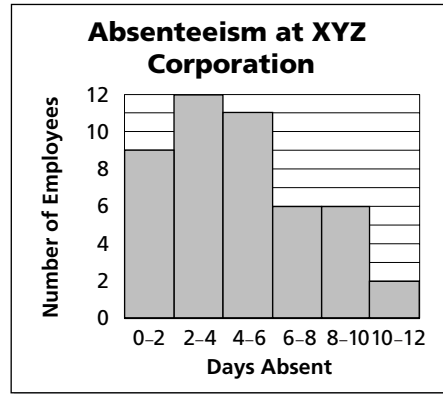**How are histograms used to display data?**

Include the following in your answer:
- the advantage of the histogram over the frequency table, and
- a histogram depicting the distribution of the mean scores on the SAT mathematics exam.

For Exercises 24 and 25, use the information in the graph.

24. How many employees are represented in the graph?

   Ⓐ 38   Ⓑ 40

   Ⓒ 46   Ⓓ 48

25. In which measurement class is the median of the data located?

   Ⓐ 2–4   Ⓑ 4–6

   Ⓒ 6–8   Ⓓ 8–10

Absenteeism at XYZ Corporation

Graphing Calculator

HISTOGRAMS   You can use a graphing calculator to create histograms. On a TI-83 Plus, enter the data in **L1**. In the **STAT PLOT** menu, turn on Plot 1 and select the histogram. Define the viewing window and press **GRAPH**.

Use a graphing calculator to create a histogram for each set of data.

26. 5, 5, 6, 7, 9, 4, 10, 12, 13, 8, 15, 16, 13, 8

27. 12, 14, 25, 30, 11, 35, 41, 47, 13, 18, 58, 59, 42, 13, 18

28. 124, 83, 81, 130, 111, 92, 178, 179, 134, 92, 133, 145, 180, 144

29. 2.2, 2.4, 7.5, 9.1, 3.4, 5.1, 6.3, 1.8, 2.8, 3.7, 8.6, 9.5, 3.6, 3.7, 5.0

Maintain Your Skills

Mixed Review

If $A = \begin{bmatrix} -2 & 3 & 7 \\ 0 & -4 & 6 \\ 1 & -5 & 4 \end{bmatrix}$, $B = \begin{bmatrix} -8 & 1 & -1 \\ 2 & 3 & -7 \end{bmatrix}$, and $C = \begin{bmatrix} 7 & -5 & 2 \\ 0 & 0 & 3 \\ -1 & 4 & 6 \end{bmatrix}$, find each sum, difference, or product. If the sum or difference does not exist, write *impossible*. *(Lesson 13-2)*

30. $A + B$ **31.** $C - A$ **32.** $2B$ **33.** $-5A$

34. **MANUFACTURING**   Every 15 minutes, a CD player is taken off the assembly line and tested. State whether this sample is *unbiased* (random) or *biased*. If unbiased, classify the sample as *simple*, *stratified*, or *systematic*. If biased, classify as *convenience* or *voluntary response*. *(Lesson 13-1)*

Find each quotient. Assume that no denominator has a value of 0.   *(Lesson 12-4)*

35. $\dfrac{s}{s+7} \div \dfrac{s-5}{s+7}$

36. $\dfrac{2m^2 + 7m - 15}{m+2} \div \dfrac{2m-3}{m^2 + 5m + 6}$

Solve each equation. Check your solution.   *(Lesson 11-3)*

37. $\sqrt{y+3} + 5 = 9$ **38.** $\sqrt{x-2} = x - 4$ **39.** $13 = \sqrt{2w-5}$

Getting Ready for the Next Lesson

PREREQUISITE SKILL   Find the median for each set of data.
*(To review **median**, see pages 818 and 819.)*

40. 2, 4, 7, 9, 12, 15   **41.** 10, 3, 17, 1, 8, 6, 12, 15

42. 7, 19, 9, 4, 7, 2   **43.** 2.1, 7.4, 13.9, 1.6, 5.21, 3.901
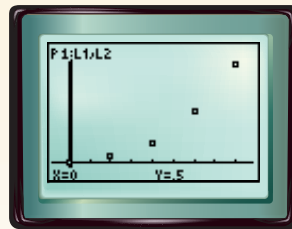
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Curve Fitting

If there is a constant increase or decrease in data values, there is a linear trend. If the values are increasing or decreasing more and more rapidly, there may be a quadratic or exponential trend. The curvature of a quadratic trend tends to appear more gradual. Below are three scatter plots, each showing a different trend.
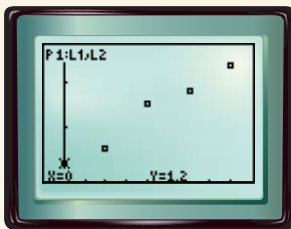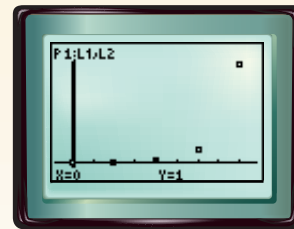
Linear Trend	**Quadratic Trend**	**Exponential Trend**

With a TI-83 Plus, you can use the LinReg, QuadReg, and ExpReg functions to find the appropriate regression equation that best fits the data.

FARMING   A study is conducted in which groups of 25 corn plants are given a different amount of fertilizer and the gain in height after a certain time is recorded. The table below shows the results.

Fertilizer (mg)	0	20	40	60	80
Gain in Height (in.)	6.48	7.35	8.73	9.00	8.13

Step 1 *Make a scatter plot.*

• Enter the fertilizer in L1 and the height in L2.

   KEYSTROKES: *Review entering a list on page 204.*

• Use **STAT PLOT** to graph the scatter plot.

   KEYSTROKES: *Review statistical plots on page 204. Use* ZOOM *9 to graph.*
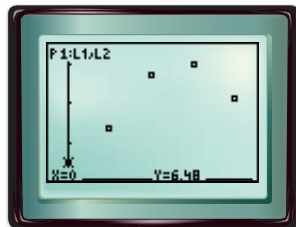
[−8, 88] scl: 5 by [6.0516, 9.4284] scl: 1

The graph appears to be a quadratic regression.

Step 2 *Find the quadratic regression equation.*

• Select **QuadReg** on the STAT CALC menu.

   KEYSTROKES: STAT ▶ 5 ENTER

The equation is in the form $y = ax^2 + bx + c$.
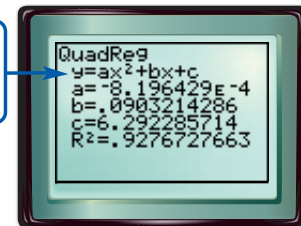
The equation is about $y = -0.0008x^2 + 0.1x + 6.3$.

R^2 is the **coefficient of determination**. The closer R^2 is to 1, the better the model. To choose a quadratic or exponential model, fit both and use the one with the R^2 value closer to 1.
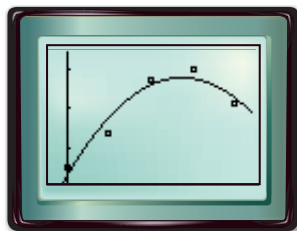
CONTENTS

Graphing Calculator Investigation

Step 3 *Graph the quadratic regression equation.*

- Copy the equation to the Y= list and graph.

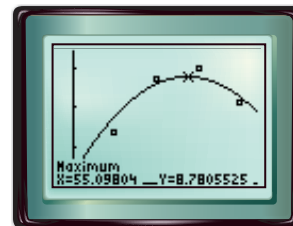 **KEYSTROKES:** [Y=] [VARS] 5 ▶ ▶ 1
  [ZOOM] 9

Step 4 *Predict using the equation.*

- Find the amount of fertilizer that produces the maximum gain in height.

On average, about 55 milligrams of the fertilizer produces the maximum gain.

Exercises

Plot each set of data points. Determine whether to use a *linear*, *quadratic*, or *exponential* regression equation. State the coefficient of determination.

1.

x	y
0.0	2.98
0.2	1.46
0.4	0.90
0.6	0.51
0.8	0.25
1.0	0.13

2.

x	y
1	25.9
2	22.2
3	20.0
4	19.3
5	18.2
6	15.9

3.

x	y
10	35
20	50
30	70
40	88
50	101
60	120

4.

x	y
1	3.67
3	5.33
5	6.33
7	5.67
9	4.33
11	2.67

TECHNOLOGY   The cost of cellular phone use is expected to decrease. For Exercises 5–9, use the graph at the right.

5. Make a scatter plot of the data.

6. Find an appropriate regression equation, and state the coefficient of determination.

7. Use the regression equation to predict the expected cost in 2004.

8. Do you believe that your regression equation is appropriate for a year beyond the range of data, such as 2020? Explain.

9. What model may be more appropriate for predicting cost beyond 2003?

USA TODAY Snapshots®

Cheaper wireless talk

Cheaper digital networks and more competition are expected to cut the cost of wireless phone use. Per-minute average in 1998 and projected cost in the next five years:

33¢ — 1998
28¢ — 1999
25¢ — 2000
23¢ — 2001
22¢ — 2002
20¢ — 2003

Source: The Strategis Group

By Anne R. Carey and Marcy E. Mullins, USA TODAY

CONTENTS

Measures of Variation

What You'll Learn

- Find the range of a set of data.
- Find the quartiles and interquartile range of a set of data.

Vocabulary

- range
- measures of variation
- quartiles
- lower quartile
- upper quartile
- interquartile range
- outlier

How is variation used in weather?

The average monthly temperatures for three U.S. cities are given. Which city shows the greatest change in monthly highs?

To answer this question, find the difference between the greatest and least values in each data set.

Buffalo: $80.2 - 30.2 = 50.0$
Honolulu: $88.7 - 80.1 = 8.6$
Tampa: $90.2 - 69.8 = 20.4$

Buffalo shows the greatest change.

Average Monthly High Temperatures (°F)

Month	Buffalo	Honolulu	Tampa
January	30.2	80.1	69.8
February	31.6	80.5	71.4
March	41.7	81.6	76.6
April	54.2	82.8	81.7
May	66.1	84.7	87.2
June	75.3	86.5	89.5
July	80.2	87.5	90.2
August	77.9	88.7	90.2
September	70.8	88.5	89.0
October	59.4	86.9	84.3
November	47.1	84.1	77.7
December	35.3	81.2	72.1

Source: www.stormfax.com

RANGE The difference between the greatest and the least monthly high temperatures is called the **range** of the temperatures.

Key Concept   Definition of Range

The range of a set of data is the difference between the greatest and the least values of the set.

Study Tip

Look Back
To review **mean**, **median**, and **mode**, see pages 818 and 819.

The mean, median, and mode describe the central tendency of a set of data. The range of a set of data is a measure of the spread of the data. Measures that describe the spread of the values in a set of data are called **measures of variation**.

Example 1 Find the Range

HOCKEY The number of wins for each team in the Eastern Conference of the NHL for the 1999–2000 season are listed below. Find the range of the data.

Team	Wins	Team	Wins	Team	Wins
Atlanta	14	Montreal	35	Philadelphia	45
Boston	24	New Jersey	45	Pittsburgh	37
Buffalo	35	N.Y. Islanders	24	Tampa Bay	19
Carolina	37	N.Y. Rangers	29	Toronto	45
Florida	43	Ottawa	41	Washington	44

Source: *The World Almanac*

The greatest number of wins is 45, and the least number of wins is 14.
Since $45 - 14 = 31$, the range of the number of wins is 31.
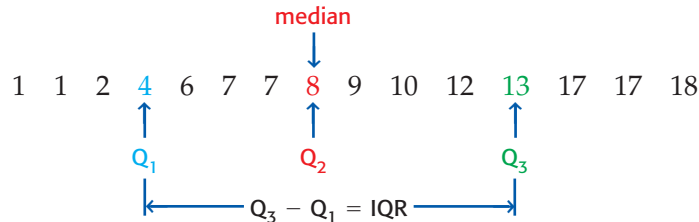
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Study Tip

Reading Math
The abbreviations LQ and UQ are often used to represent the lower quartile and upper quartile, respectively.

QUARTILES AND INTERQUARTILE RANGE   In a set of data, the **quartiles** are values that separate the data into four equal subsets, each containing one fourth of the data. Statisticians often use Q_1, Q_2, and Q_3 to represent the three quartiles. Remember that the median separates the data into two equal parts. Q_2 is the median. Q_1 is the **lower quartile**. It divides the lower half of the data into two equal parts. Likewise Q_3 is the **upper quartile**. It divides the upper half of the data into two equal parts. The difference between the upper and lower quartiles is the **interquartile range** (IQR).

median
↓

1   1   2   **4**   6   7   7   **8**   9   10   12   **13**   17   17   18
        ↑   ↑   ↑
       Q_1 Q_2 Q_3

←———— $Q_3 - Q_1$ = IQR ————→

Key Concept   *Definition of Interquartile Range*

The difference between the upper quartile and the lower quartile of a set of data is called the interquartile range. It represents the middle half, or 50%, of the data in the set.

Example **2** *Find the Quartiles and the Interquartile Range*

GEOGRAPHY   The areas of the original 13 states are listed in the table. Find the median, the lower quartile, the upper quartile, and the interquartile range of the areas.
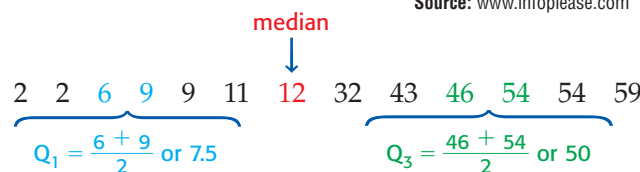
Explore   You are given a table with the areas of the original 13 states. You are asked to find the median, the lower quartile, the upper quartile, and the interquartile range.

Plan   First, list the areas from least to greatest. Then find the median of the data. The median will divide the data into two sets of data. To find the upper and lower quartiles, find the median of each of these sets of data. Finally, subtract the lower quartile from the upper quartile to find the interquartile range.

State	Area (thousand square miles)
Connecticut	6
Delaware	2
Georgia	59
Maryland	12
Massachusetts	11
New Hampshire	9
New Jersey	9
New York	54
North Carolina	54
Pennsylvania	46
Rhode Island	2
South Carolina	32
Virginia	43

Source: www.infoplease.com

Solve

median
↓

2   2   **6**   **9**   9   11   **12**   32   43   **46**   **54**   54   59

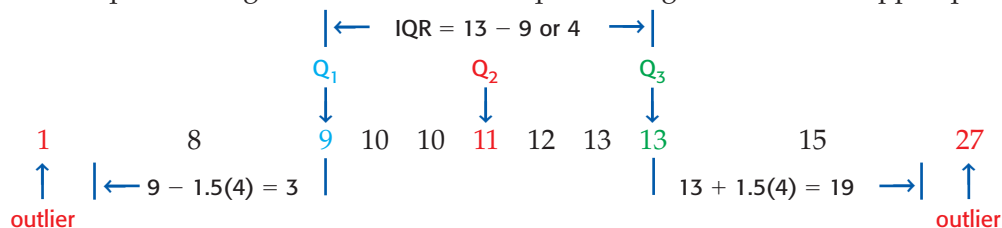$Q_1 = \dfrac{6 + 9}{2}$ or 7.5 $Q_3 = \dfrac{46 + 54}{2}$ or 50

The median is 12 thousand square miles.

The lower quartile is 7.5 thousand square miles, and the upper quartile is 50 thousand square miles.

The interquartile range is 50 − 7.5 or 42.5 thousand square miles.

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Examine Check to make sure that the numbers are listed in order. Since 7.5, 12, and 50 divide the data into four equal parts, the lower quartile, median, and upper quartile are correct.

In a set of data, a value that is much less or much greater than the rest of the data is called an **outlier**. An outlier is defined as any element of a set of data that is at least 1.5 interquartile ranges less than the lower quartile or greater than the upper quartile.

$$\leftarrow \quad \text{IQR} = 13 - 9 \text{ or } 4 \quad \rightarrow$$

		Q_1			Q_2			Q_3		
1	8	9	10	10	11	12	13	13	15	27

↑ outlier $\leftarrow 9 - 1.5(4) = 3$ $13 + 1.5(4) = 19 \rightarrow$ ↑ outlier

Example 3 Identify Outliers

Identify any outliers in the following set of data.

Stem	Leaf
1	[2 2 7
2	3 3 3 4 4 5 6] [6 8 8 9
3	0 1 4 6
4	0 6] $1 \mid 2 = 12$

Step 1 Find the quartiles.

The brackets group the values in the lower half and the values in the upper half. The boxes are used to find the lower quartile and the upper quartile.

$$Q_1 = \frac{23 + 23}{2} \text{ or } 23 \qquad\qquad Q_3 = \frac{30 + 31}{2} \text{ or } 30.5$$

Step 2 Find the interquartile range.

The interquartile range is $30.5 - 23$ or 7.5.

Step 3 Find the outliers, if any.

An outlier must be 1.5(7.5) less than the lower quartile, 23, or 1.5(7.5) greater than the upper quartile, 30.5.

$$23 - 1.5(7.5) = 11.75 \qquad\qquad 30.5 + 1.5(7.5) = 41.75$$

There are no values less than 11.75. Since $46 > 41.75$, 46 is the only outlier.

Check for Understanding

Concept Check **1. OPEN ENDED** Find a counterexample for the following statement.

If the range of data set 1 is greater than the range of data set 2, then the interquartile range of data set 1 will be greater than the interquartile range of data set 2.

2. Describe how the mean is affected by an outlier.

3. FIND THE ERROR Alonso and Sonia are finding the range of this set of data: 28, 30, 32, 36, 40, 41, 43.

Alonso
43 − 28 = 15
The range is 15.

Sonia
The range is all numbers
between 28 and 43, inclusive.

Who is correct? Explain your reasoning.

CONTENTS

Guided Practice

Find the range, median, lower quartile, upper quartile, and interquartile range of each set of data. Identify any outliers.

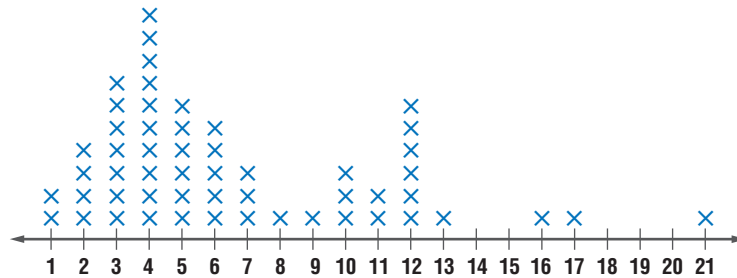4. 85, 77, 58, 69, 62, 73, 25, 82, 67, 77, 59, 75, 69, 76

5.

Stem	Leaf
7	3 7 8
8	0 0 3 5 7
9	4 6 8
10	0 1 8
11	1 9 $7|3 = 7.3$

Application

LITTLE LEAGUE   **For Exercises 6–10, use the following information.**
The number of runs scored by the winning team in the Little League World Series each year from 1947 to 2000 are given in the line plot below.

Source: *The World Almanac*

6. What is the range of the data?

7. What is the median of the data?

8. What is the lower quartile and upper quartile of the data?

9. What is the interquartile range of the data?

10. Name any outliers.

Practice and Apply

Find the range, median, lower quartile, upper quartile, and interquartile range of each set of data. Identify any outliers.

11. 85, 77, 58, 69, 62, 73, 55, 82, 67, 77, 59, 92, 75

12. 28, 42, 37, 31, 34, 29, 44, 28, 38, 40, 39, 42, 30

13. 30.8, 29.9, 30.0, 31.0, 30.1, 30.5, 30.7, 31.0

14. 2, 3.4, 5.3, 3, 1, 3.2, 4.9, 2.3

15.

Stem	Leaf
5	3 6 8
6	5 8
7	0 3 7 7 9
8	1 4 8 8 9
9	9 $5|3 = 53$

16.

Stem	Leaf
19	3 5 5
20	2 2 5 8
21	5 8 8 9 9 9
22	0 1 7 8 9
23	2 $19|3 = 193$

17.

Stem	Leaf
5	0 3 7 9
6	1 3 4 5 5 6
7	1 5 6 6 9
8	1 2 3 5 8
9	2 5 6 9
10	
11	7 $5|0 = 5.0$

18.

Stem	Leaf
0	0 2 3
1	1 7 9
2	2 3 5 6
3	3 4 4 5 9
4	0 7 8 8
5	
6	8 $0|2 = 0.2$

CONTENTS

NATIONAL PARKS For Exercises 19–23, use the graph at the right.

19. What is the range of the visitors per month?

20. What is the median number of visitors per month?

21. What are the lower quartile and the upper quartile of the data?

22. What is the interquartile range of the data?

23. Name any outliers.

Average Number of Visitors to Yosemite
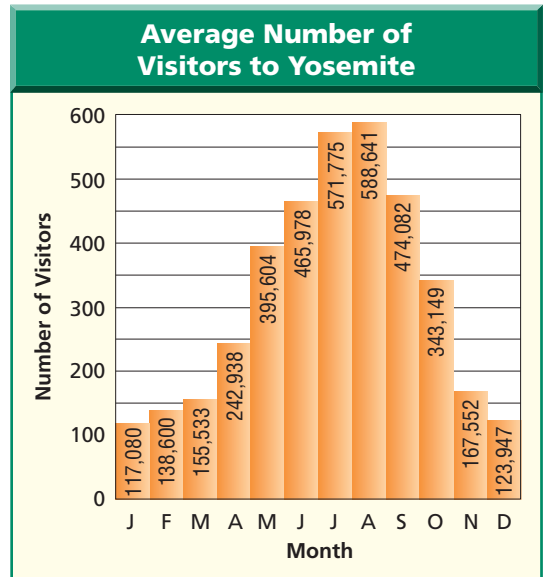
Source: *USA TODAY*

NUTRITION For Exercises 24–28, use the following table.

Calories for One Serving of Vegetables					
Vegetable	Calories	Vegetable	Calories	Vegetable	Calories
Asparagus	14	Carrots	28	Lettuce	9
Avocado	304	Cauliflower	10	Onion	60
Bell pepper	20	Celery	17	Potato	89
Broccoli	25	Corn	66	Spinach	9
Brussels sprouts	60	Green beans	30	Tomato	35
Cabbage	17	Jalapeno peppers	13	Zucchini	17

Source: *Vitality*

24. What is the range of the data?   25. What is the median of the data?

26. What are the lower quartile and the upper quartile of the data?

27. What is the interquartile range of the data?

28. Identify any outliers.

BRIDGES For Exercises 29–33, use the following information and the double stem-and-leaf plot at the right.
The main span of cable-stayed bridges and of steel-arch bridges in the United States are given in the stem-and-leaf plot.

29. Find the ranges for each type of bridge.

30. Find the quartiles for each type of bridge.

31. Find the interquartile ranges for each type of bridge.

32. Identify any outliers.

33. Compare the ranges and interquartile ranges of the two types of bridges. What can you conclude from these statistics?

| Cable-Stayed	Stem	Steel-Arch
6 4 3	6	
9 8 6 5 1 1	7	3 8 8
0	8	0 0 2 3 4
5 2	9	0 1 1 8 8 9
8 2 0 0	10	0 3 8
2	11	0 0
8 2 0	12	0 6
5 2 0	13	
9 0	14	
	15	
3	16	5
	17	0

3|6 = 630 feet   *7|3 = 730 feet*

Source: *The World Almanac*

34. CRITICAL THINKING Trey measured the length of each classroom in his school. He then calculated the range, median, lower quartile, upper quartile, and interquartile range of the data. After his calculations, he discovered that the tape measure he had used started at the 2-inch mark instead of at the 0-inch mark. All of his measurements were 2 inches greater than the actual lengths of the rooms. How will the values that Trey calculated change? Explain your reasoning.

35. WRITING IN MATH Answer the question that was posed at the beginning of the lesson.

How is variation used in weather?

Include the following in your answer:
- the meaning of the range and interquartile range of temperatures for a city, and
- the average highs for your community with the appropriate measures of variation.
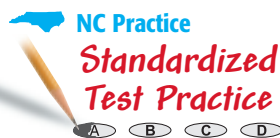
36. What is the range of the following set of data?
53, 57, 62, 48, 45, 65, 40, 42, 55

Ⓐ 11   Ⓑ 25   Ⓒ 53   Ⓓ 65

37. What is the median of the following set of data?
7, 8, 14, 3, 2, 1, 24, 18, 9, 15

Ⓐ 8.5   Ⓑ 10.1   Ⓒ 11.5   Ⓓ 23

Maintain Your Skills

Mixed Review

38. Create a histogram to represent the following data. *(Lesson 13-3)*
36, 43, 61, 45, 37, 41, 32, 46, 60, 38, 35, 64, 46, 47, 30, 38, 48, 39

State the dimensions of each matrix. Then identify the position of the circled element in each matrix. *(Lesson 13-2)*

39. $[\text{⑤} \quad -3 \quad 6]$

40. $\begin{bmatrix} 3 & 1 \\ 2 & 9 \\ 4 & ③ \end{bmatrix}$

41. $\begin{bmatrix} 4 & 2 & -1 & 3 \\ 5 & ⑨ & 0 & 2 \end{bmatrix}$

Simplify each rational expression. State the excluded values of the variables. *(Lesson 12-2)*

42. $\dfrac{15a}{39a^2}$

43. $\dfrac{t - 3}{t^2 - 7t + 12}$

44. $\dfrac{m - 3}{m^2 - 9}$

Getting Ready for the Next Lesson

PREREQUISITE SKILL Graph each set of numbers on a number line.
*(To review **number lines**, see Lesson 2-1.)*

45. {4, 7, 8, 10, 11}   **46.** {13, 17, 22, 23, 27}   **47.** {30, 35, 40, 50, 55}

## Practice Quiz 2   Lessons 13-3 and 13-4

For Exercises 1–2, use the histogram at the right. *(Lesson 13-3)*

1. In what measurement class does the median occur?

2. Describe the distribution of the data.

For Exercises 3–5, use the following set of data. *(Lesson 13-4)*
1050, 1175, 835, 1075, 1025, 1145, 1100,
1125, 975, 1005, 1125, 1095, 1075, 1055

3. Find the range of the data.

4. Find the median, the lower quartile, the upper quartile, and interquartile range of the data.

5. Identify any outliers of the data.
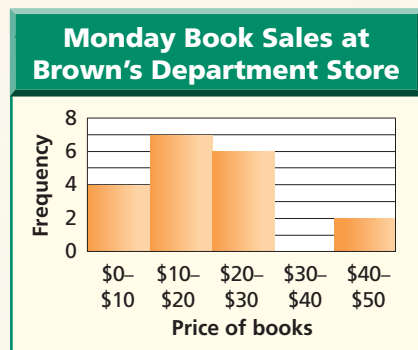
Monday Book Sales at Brown's Department Store

CONTENTS

13-5 Box-and-Whisker Plots

- Organize and use data in box-and-whisker plots.
- Organize and use data in parallel box-and-whisker plots.

Vocabulary

- box-and-whisker plot
- extreme values

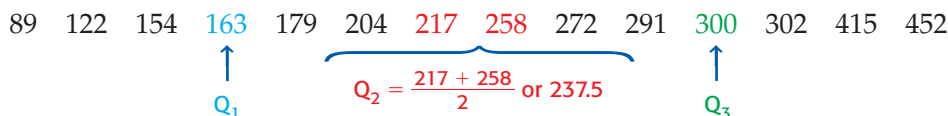**How** are box-and-whisker plots used to display data?

Everyone should eat a number of calcium-rich foods each day. Selected foods and the amount of calcium in a serving are listed in the table. To create a box-and-whisker plot of the data, you need to find the quartiles of the data.
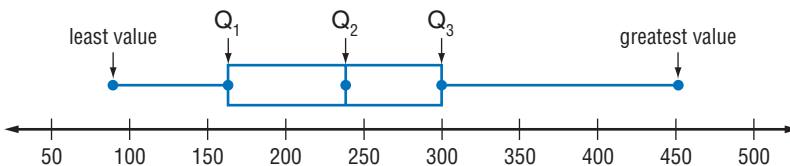
Calcium-Rich Foods

Food (serving size)	Calcium (milligrams)
Plain Yogurt, Nonfat (8 oz)	452
Plain Yogurt, Low-fat (8 oz)	415
Skim Milk (8 oz)	302
1% Milk (8 oz)	300
Whole Milk (8 oz)	291
Swiss Cheese (1 oz)	272
Tofu (4 oz)	258
Sardines (2 oz)	217
Cheddar Cheese (1 oz)	204
Collards (4 oz)	179
American Cheese (1 oz)	163
Frozen Yogurt with Fruit (4 oz)	154
Salmon (2 oz)	122
Broccoli (4 oz)	89

Source: *Vitality*

89 122 154 163 179 204 217 258 272 291 300 302 415 452

$$Q_1 \qquad Q_2 = \frac{217 + 258}{2} \text{ or } 237.5 \qquad Q_3$$

This information can be displayed on a number line as shown below.

Study Tip

Reading Math
Box-and-whisker plots are sometimes called *box plots*.

BOX-AND-WHISKER PLOTS Diagrams such as the one above are called **box-and-whisker plots**. The length of the box represents the interquartile range. The line inside the box represents the median. The lines or *whiskers* represent the values in the lower fourth of the data and the upper fourth of the data. The bullets at each end are the **extreme values**. In the box-and-whisker plot above, the least value (LV) is 89, and the greatest value (GV) is 452.

If a set of data has outliers, these data points are represented by bullets. The whisker representing the lower data is drawn from the box to the least value that is not an outlier. The whisker representing the upper data is drawn from the box to the greatest value that is not an outlier.

Example 1 **Draw a Box-and-Whisker Plot**

ECOLOGY The amount of rain in Florida from January to May is crucial to its ecosystems. The following is a list of the number of inches of rain in Florida during this crucial period for the years 1990 to 2000.

14.03, 30.11, 16.03, 19.61, 18.15, 16.34, 20.43, 18.46, 22.24, 12.70, 8.25

a. Draw a box-and-whisker plot for these data.

Step 1 **Determine the quartiles and any outliers.**

Order the data from least to greatest. Use this list to determine the quartiles.

8.25, 12.70, 14.03, 16.03, 16.34, 18.15, 18.46, 19.61, 20.43, 22.24, 30.11

$$Q_1 \qquad\qquad Q_2 \qquad\qquad Q_3$$

Determine the interquartile range.

IQR = 20.43 − 14.03 or 6.4

Check to see if there are any outliers.

14.03 − 1.5(6.4) = 4.43   20.43 + 1.5(6.4) = 30.03

Any numbers less than 4.43 or greater than 30.03 are outliers. The only outlier is 30.11.

Step 2 **Draw a number line.**

Assign a scale to the number line that includes the extreme values. Above the number line, place bullets to represent the three quartile points, any outliers, the least number that is *not* an outlier, and the greatest number that is *not* an outlier.
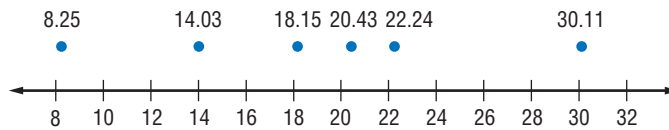
Step 3 **Complete the box-and-whisker plot.**

Draw a box to designate the data between the upper and lower quartiles. Draw a vertical line through the point representing the median. Draw a line from the lower quartile to the least value that is *not* an outlier. Draw a line from the upper quartile to the greatest value that is *not* an outlier.
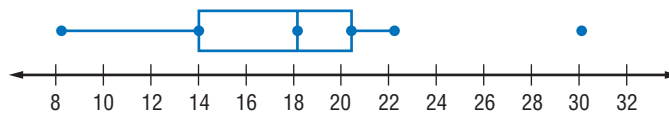
b. What does the box-and-whisker plot tell about the data?

Notice that the whisker and the box for the top half of the data is shorter than the whisker and box for the lower half of the data. Therefore, except for the outlier, the upper half of the data are less spread out than the lower half of the data.

PARALLEL BOX-AND-WHISKER PLOTS Two sets of data can be compared by drawing parallel box-and-whisker plots such as the one shown below.
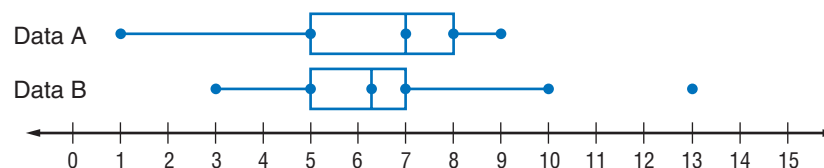
More About...

Ecology

The ecosystem of the Everglades in Florida is unique. The Everglades National Park is a subtropical swamp area covering 1,506,499 acres.

Source: *World Book Encyclopedia*

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738 Chapter 13 Statistics

CONTENTS

Example 2 **Draw Parallel Box-and-Whisker Plots**

WEATHER Jalisa Thompson has job offers in Fresno, California, and Brownsville, Texas. Since she likes both job offers, she decides to compare the temperatures of each city.

Average Monthly High Temperatures (°F)												
Month	Jan.	Feb.	March	April	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
Fresno	54.1	61.7	66.6	75.1	84.2	92.7	98.6	96.7	90.1	79.7	64.7	53.7
Brownsville	68.9	72.2	78.4	84.0	87.8	91.0	93.3	93.6	90.4	85.3	78.3	71.7

Source: www.stormfax.com

a. **Draw a parallel box-and-whisker plot for the data.**

Determine the quartiles and outliers for each city.

Fresno

53.7, 54.1, 61.7, 64.7, 66.6, 75.1, 79.7, 84.2, 90.1, 92.7, 96.7, 98.6

$Q_1 = 63.2$ $Q_2 = 77.4$ $Q_3 = 91.4$

Brownsville

68.9, 71.7, 72.2, 78.3, 78.4, 84.0, 85.3, 87.8, 90.4, 91.0, 93.3, 93.6

$Q_1 = 75.25$ $Q_2 = 84.65$ $Q_3 = 90.7$

Neither city has any outliers.

Draw the box-and-whisker plots using the same number line.
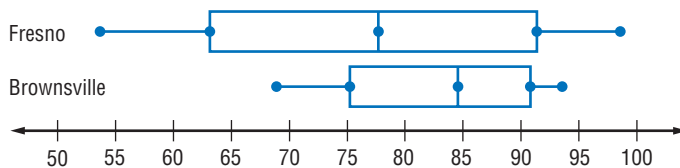
b. **Use the parallel box-and-whisker plots to compare the data.**

The range of temperatures in Fresno is much greater than in Brownsville. Except for the fourth quartile, Brownville's average temperatures appear to be as high or higher than Fresno's.

Check for Understanding

Concept Check

1. **Describe** the data represented by the box-and-whisker plot at the right. Include the extreme values, the quartiles, and any outliers.

2. **Explain** how to determine the scale of the number line in a box-and-whisker plot.

3. **OPEN ENDED** Write a set of data that could be represented by the box-and-whisker plot at the right.
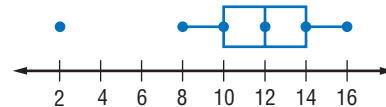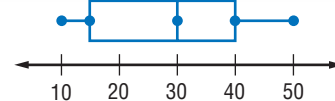
Guided Practice

Draw a box-and-whisker plot for each set of data.

4. 30, 28, 24, 24, 22, 22, 21, 17, 16, 15

5. 64, 69, 65, 71, 66, 66, 74, 67, 68, 67

www.algebra1.com/extra_examples/nc

CONTENTS

Draw a parallel box-and-whisker plot for each pair of data. Compare the data.

6. A: 22, 18, 22, 17, 32, 24, 31, 26, 28
   B: 28, 30, 45, 23, 24, 32, 30, 27, 27

7. A: 8, 15.5, 14, 14, 24, 19, 16.7, 15, 11.4, 16
   B: 18, 14, 15.8, 9, 12, 16, 20, 16, 13, 15

Application **CHARITY** **For Exercises 8 and 9, use the information in the table below.**

Top Ten Charities	
Charity	**Private Contributions (millions)**
Salvation Army	$1397
YMCA of the U.S.A.	$693
American Red Cross	$678
American Cancer Society	$620
Fidelity Investments Charitable Gift Fund	$573
Lutheran Services in America	$559
United Jewish Communities	$524
America's Second Harvest	$472
Habitat for Humanity International	$467
Harvard University	$452

Source: *The Chronicle of Philanthropy*

8. Make a box-and-whisker for the data.

9. Write a brief description of the data distribution.

Practice and Apply

For Exercises 10–13, use the box-and-whisker plot at the right.

10. What is the range of the data?

11. What is the interquartile range of the data?

12. What fractional part of the data is less than 90?

13. What fractional part of the data is greater than 95?
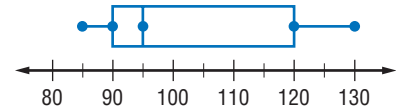
Draw a box-and-whisker plot for each set of data.

14. 15, 8, 10, 1, 3, 2, 6, 5, 4, 27, 1

15. 20, 2, 12, 5, 4, 16, 17, 7, 6, 16, 5, 0, 5, 30

16. 4, 1, 1, 1, 10, 15, 4, 5, 27, 5, 14, 10, 6, 2, 2, 5, 8

17. 51, 27, 55, 54, 69, 60, 39, 46, 46, 53, 81, 23

18. 15.1, 9.0, 8.5, 5.8, 6.2, 8.5, 10.5, 11.5, 8.8, 7.6

19. 1.3, 1.2, 14, 1.8, 1.6, 5.7, 1.3, 3.7, 3.3, 2, 1.3, 1.3, 7.7, 8.5, 2.2

For Exercises 20–23, use the parallel box-and-whisker plot at the right.

20. Which set of data contains the least value?

21. Which set of data contains the greatest value?

22. Which set of data has the greatest interquartile range?

23. Which set of data has the greatest range?
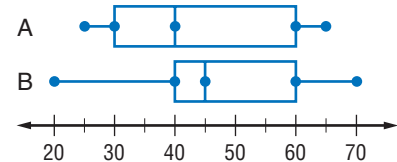
Draw a parallel box-and-whisker plot for each pair of data. Compare the data.

24. A: 15, 17, 22, 28, 32, 40, 16, 24, 26, 38, 19
   B: 24, 32, 25, 27, 37, 29, 30, 30, 28, 31, 27

CONTENTS

25. A: 50, 45, 47, 55, 51, 58, 49, 51, 51, 48, 47
B: 40, 41, 48, 39, 41, 41, 38, 37, 35, 37, 45

26. A: 1.5, 3.8, 4.2, 3.5 4.1, 4.4, 4.1, 4.0, 4.0, 3.9
B: 6.8, 4.2, 7.6, 5.5, 12.2, 6.7, 7.1, 4.8

27. A: 4.4, 4.5, 4.6, 4.5, 4.4, 4.4, 4.1, 4.9, 2.9
B: 5.1, 4.9, 4.2, 3.9, 4.5, 4.1, 4.3, 4.5, 5.2

PROFESSIONAL SPORTS For Exercises 28 and 29, use the table at the right.

28. Draw a box-and-whisker plot for the data.

29. What does the box-and-whisker plot tell about the data?

Professional Athletes

Professional Sport	Average Length of Career (years)
Bowling	17
Surfing	10
Hockey	5.5
Baseball	4.5
Basketball	4.5
Tennis	4
Football	3.5
Boxing	3.5

Source: *Men's Health Fitness Special*

RACING For Exercises 30 and 31, use the following list of earnings in thousands from the November 2000 NAPA 500 NASCAR Race at the Atlanta Motor Speedway.

$181, $100, $98, $89, $76, $58, $60; $58; $55, $57, $54, $64, $44, $39, $66, $52, $56, $38, $56, $51, $49, $38, $50, $48, $48, $40, $36, $36, $39, $36, $47, $36, $47, $38, $35, $46, $35, $55, $46, $55, $45, $43, $35

Source: *USA TODAY*

30. Draw a box-and-whisker plot for the data. Identify any outliers.

31. Determine whether the top half of the data or the bottom half of the data are more dispersed. Explain.
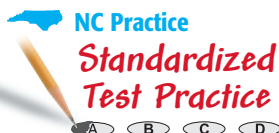
Life Expectancy

A newborn resident of the United States has a life expectancy of 77 years, while a newborn resident of Canada has a life expectancy of 79 years.

Source: UNICEF

LIFE EXPECTANCY For Exercises 32–35, use the box-and-whisker plot depicting the UNICEF life expectancy data for 171 countries.
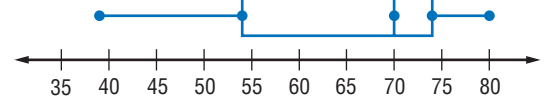
32. Estimate the range and the interquartile range.

33. Determine whether the top half of the data or the bottom half of the data are more dispersed. Explain.

34. State three different intervals of ages that contain half the data.

35. Jamie claims that the number of data values is greater in the interval 54 years to 70 years than the number of data values in the interval 70 years to 74 years. Is Jamie correct? Explain.

SOCCER For Exercises 36–38, use the following list of top 50 lifetime scores for all players in Division 1 soccer leagues in the United States from 1922 to 1999.

253, 223, 193, 189, 152, 150, 138, 137, 135, 131, 131, 129, 128, 126, 124, 119, 118, 108, 107, 102, 101, 100, 96, 92, 87, 83, 82, 81, 80, 78, 78, 76, 74, 74, 73, 73, 72, 71, 69, 68, 67, 65, 64, 63, 63, 63, 62, 61, 61, 61

Source: www.internetsoccer.com

36. Draw a box-and-whisker plot for the data.

37. Draw a histogram to represent the data.

38. Compare and contrast the box-and-whisker plot and the histogram.

39. CRITICAL THINKING   Write a set of data that could be represented by the box-and-whisker plot at the right.
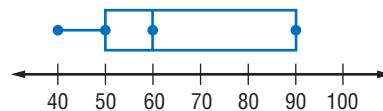
40. WRITING IN MATH   Answer the question that was posed at the beginning of the lesson.

**How are box-and-whisker plots used to display data?**

Include the following in your answer:

- a sample of a box-and-whisker plot showing what each part of the plot represents, and
- a box-and-whisker plot representing data found in a newspaper or magazine.

NC Practice
Standardized Test Practice
Ⓐ Ⓑ Ⓒ Ⓓ

For Exercises 41 and 42, use the box-and-whisker plot below.

41. What is the median of the data?

Ⓐ 0
Ⓒ 25
Ⓑ 10
Ⓓ 45
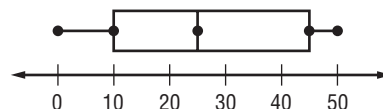
42. Which interval represents 75% of the data?

Ⓐ 0–25
Ⓑ 10–45
Ⓒ 25–50
Ⓓ 0–45

Maintain Your Skills

Mixed Review **For Exercises 43 and 44, use the following data.**

13, 32, 45, 45, 54, 55, 58, 67, 82, 93

43. Find the range, median, lower quartile, upper quartile, and interquartile range of the data. Identify any outliers.   *(Lesson 13-4)*

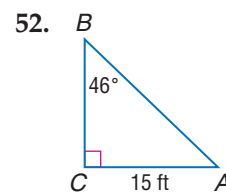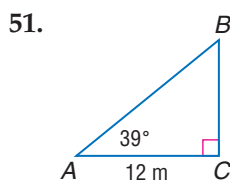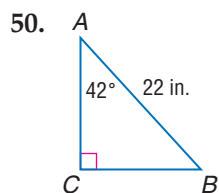**44.** Create a histogram to represent the data.   *(Lesson 13-3)*

Find each sum or difference.   *(Lesson 12-7)*

45. $\dfrac{3}{y-3} - \dfrac{y}{y+4}$

46. $\dfrac{2}{r+3} + \dfrac{3}{r-2}$

47. $\dfrac{w}{5w+2} - \dfrac{4}{15w+6}$

Find each product. Assume that no denominator has a value of 0.   *(Lesson 12-3)*

48. $\dfrac{7a^2}{5} \cdot \dfrac{15}{14a}$

49. $\dfrac{6r+3}{r+6} \cdot \dfrac{r^2+9r+18}{2r+1}$

Solve each right triangle. State the side length to the nearest tenth and the angle measures to the nearest degree.   *(Lesson 11-7)*

50.

51.

52.

Solve each equation by completing the square. Approximate any irrational roots to the nearest tenth.   *(Lesson 10-3)*

53. $a^2 - 7a + 6 = 0$

54. $x^2 - 6x + 2 = 0$

55. $t^2 + 8t - 18 = 0$

Find each sum or difference.   *(Lesson 8-5)*

56. $(7p^2 - p - 7) - (p^2 + 11)$

57. $(3a^2 - 8) + (5a^2 + 2a + 7)$

CONTENTS

Algebra Activity

Investigating Percentiles

When data are arranged in order from least to greatest, you can describe the data using percentiles. A **percentile** is the point below which a given percent of the data lies. For example, 50% of the data falls below the median. So the median is the 50th percentile for the data.

To determine a percentile, a cumulative frequency table can be used. In a **cumulative frequency table**, the frequencies are accumulated for each item.

Collect the Data

A student's score on the SAT is one factor that some colleges consider when selecting applicants. The tables below show the raw scores from a sample math SAT test for 160 juniors in a particular school. For raw scores, the highest possible score is 800 and the lowest is 200.

Table 1: Frequency Table
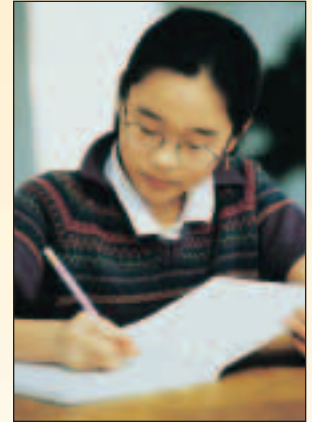
Math SAT Scores	Number of Students
200–300	2
300–400	19
400–500	44
500–600	55
600–700	32
700–800	8

Table 2: Cumulative Frequency Table

Math SAT Scores	Number of Students	Cumulative Number of Students
200–300	2	2
300–400	19	21
400–500	44	65
500–600	55	120
600–700	32	152
700–800	8	160

The data in each table can be displayed in a histogram.

Frequency Histogram

Math SAT Scores

Cumulative Frequency Histogram

Math SAT Scores
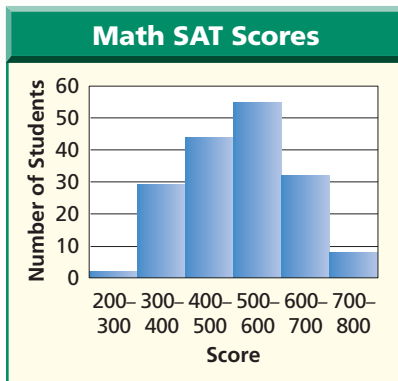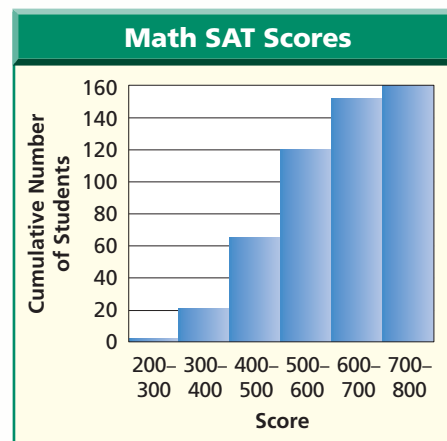
Analyze the Data

1. Examine the data in the two tables. Explain how the numbers in Column 3 of Table 2 are determined.

2. Describe the similarities and differences between the two histograms.

3. Which histogram do you prefer for displaying these data? Explain your choice.

Make a Conjecture

Sometimes colleges are not interested in your raw score. They are interested in the percentile. Your percentile indicates what percent of all test-takers scored just as well or lower than you.

4. Use the histogram for Table 2. Place percentile labels on the vertical axis. For example, write 100% next to 160 and 0% next to 0. Now label 25%, 50%, and 75%. What numbers of students correspond to 25%, 50%, and 75%?
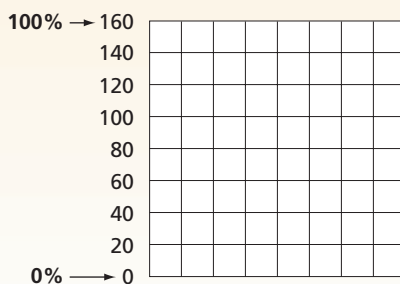
5. Suppose a college is interested in students with scores in the 90th percentile. Using the histogram, move up along the vertical axis to the 90th percentile. Then move right on the horizontal axis to find the score. What is an estimate for the score that represents the 90th percentile?

6. For a more accurate answer, use a proportion to find 90% of the total number of students. (Recall that the total number of students is 160.)

7. If a student is to be in the 90th percentile, in what interval will the score lie?

Extend the Activity

For Exercises 8–10, use the following information.

The weights of 45 babies born at a particular hospital during the month of January are shown below.

9 lb 1 oz	8 lb 2 oz	7 lb 2 oz	10 lb 0 oz	4 lb 4 oz
5 lb 0 oz	7 lb 6 oz	7 lb 8 oz	11 lb 2 oz	6 lb 1 oz
3 lb 8 oz	8 lb 0 oz	7 lb 5 oz	9 lb 15 oz	6 lb 1 oz
7 lb 10 oz	6 lb 9 oz	6 lb 15 oz	7 lb 10 oz	8 lb 0 oz
5 lb 15 oz	8 lb 3 oz	8 lb 1 oz	7 lb 12 oz	7 lb 8 oz
7 lb 7 oz	6 lb 14 oz	7 lb 13 oz	8 lb 0 oz	7 lb 14 oz
5 lb 10 oz	8 lb 5 oz	6 lb 12 oz	8 lb 8 oz	7 lb 11 oz
8 lb 15 oz	9 lb 3 oz	5 lb 14 oz	6 lb 8 oz	8 lb 8 oz
7 lb 4 oz	7 lb 10 oz	8 lb 1 oz	7 lb 8 oz	7 lb 10 oz

8. Make a cumulative frequency table for the data.

9. Make a cumulative frequency histogram for the data.

10. Find the weight for a baby in the 80th percentile.

Vocabulary and Concept Check

biased sample (p. 709)	histogram (p. 722)	range (p. 731)
box-and-whisker plot (p. 737)	interquartile range (p. 732)	row (p. 715)
census (p. 708)	lower quartile (p. 732)	sample (p. 708)
column (p. 715)	matrix (p. 715)	scalar multiplication (p. 717)
convenience sample (p. 709)	measurement classes (p. 722)	simple random sample (p. 708)
dimensions (p. 715)	measures of variation (p. 731)	stratified random sample (p. 708)
element (p. 715)	outlier (p. 733)	systematic random sample (p. 708)
extreme value (p. 737)	population (p. 708)	upper quartile (p. 732)
frequency (p. 722)	quartiles (p. 732)	voluntary response sample (p. 709)
frequency table (p.722)	random sample (p. 708)	

Choose the correct term from the list above that best completes each statement.

1. A(n) _____ is a sample that is as likely to be chosen as any other from the population.
2. Measures that describe the spread of the values in a set of data are called _____.
3. Each _____ separates a data set into four sets with equal number of members.
4. In a(n) _____, the items are selected according to a specified time or item interval.
5. A(n) _____ has a systematic error within it so that certain populations are favored.
6. In a(n) _____, the population is first divided into similar, nonoverlapping groups.
7. The _____ is found by subtracting the lower quartile from the upper quartile.
8. A(n) _____ involves only those who want to participate in the sampling.
9. An extreme value that is much less or greater than the rest of the data is a(n) _____.
10. The _____ is the difference between the greatest and least values of a data set.

Lesson-by-Lesson Review

13-1 Sampling and Bias

See pages 708–713.

Concept Summary

- Samples are used to represent a larger group called a population.
- Simple random sample, stratified random sample, and systematic random sample are types of unbiased, or random, samples.
- Convenience sample and voluntary response sample are types of biased samples.

Example   **GOVERNMENT** **To determine whether voters support a new trade agreement, 5 people from the list of registered voters in each state and the District of Columbia are selected at random. Identify the sample, suggest a population from which it was selected, and state whether the sample is *unbiased* (random) or *biased*. If unbiased, classify the sample as *simple*, *stratified*, or *systematic*. If biased, classify as *convenience* or *voluntary response*.**

Since $5 \times 51 = 255$, the sample is 255 registered voters in the United States. The population is all of the registered voters in the United States.

The sample is unbiased. It is an example of a stratified random sample.

Exercises   Identify the sample, suggest a population from which it was selected, and state whether it is *unbiased* (random) or *biased*. If unbiased, classify the sample as *simple, stratified,* or *systematic*. If biased, classify the sample as *convenience* or *voluntary response*.   *See Examples 1–3 on pages 709 and 710.*

11. **SCIENCE**   A laboratory technician needs a sample of results of chemical reactions. She selects test tubes from the first 8 experiments performed on Tuesday.

12. **CANDY BARS**   To ensure that all of the chocolate bars are the appropriate weight, every 50th bar on the conveyor belt in the candy factory is removed and weighed.

13-2 Introduction to Matrices

See pages 715–721.

Concept Summary

- A matrix can be used to organize data and make data analysis more convenient.
- Equal matrices must have the same dimensions and corresponding elements are equal.
- Matrices with the same dimensions can be added or subtracted.
- Each element of a matrix can be multiplied by a number called a scalar.

Example   If $R = \begin{bmatrix} 2 & 2 \\ -1 & 3 \end{bmatrix}$, $S = \begin{bmatrix} -1 & 3 \\ 0 & 1 \end{bmatrix}$, and $T = \begin{bmatrix} -1 \\ 0 \end{bmatrix}$, find each sum. If it does not exist, write *impossible*.

a. $R + S$

$$R + S = \begin{bmatrix} 2 & 2 \\ -1 & 3 \end{bmatrix} + \begin{bmatrix} -1 & 3 \\ 0 & 1 \end{bmatrix}$$
$$= \begin{bmatrix} 2 + (-1) & 2 + 3 \\ -1 + 0 & 3 + 1 \end{bmatrix}$$
$$= \begin{bmatrix} 1 & 5 \\ -1 & 4 \end{bmatrix}$$

b. $S + T$

$$S + T = \begin{bmatrix} -1 & 3 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} -1 \\ 0 \end{bmatrix}$$

Since S is a 2×2 matrix and T is a 2×1 matrix, the matrices do not have the same dimensions. Therefore, it is impossible to add these matrices.

Exercises   If $A = \begin{bmatrix} 1 & 3 & -1 \\ 2 & 0 & 4 \\ -1 & -1 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 1 & -3 \\ 2 & 3 & -1 \\ -1 & -2 & 0 \end{bmatrix}$, $C = \begin{bmatrix} 3 & -2 \\ 1 & 4 \end{bmatrix}$, and $D = \begin{bmatrix} 2 & 1 \\ -2 & 0 \end{bmatrix}$, find each sum, difference, or product. If the sum or difference does not exist, write *impossible*.   *See Examples 3 and 4 on pages 716 and 717.*

13. $A + B$
14. $3B$
15. $-2D$
16. $C - D$
17. $C + D$
18. $B + C$
19. $5A$
20. $A - D$
21. $C + 3D$
22. $2A - B$

13-3 Histograms

See pages 722–728.

Concept Summary

- A histogram can illustrate the information in a frequency table.
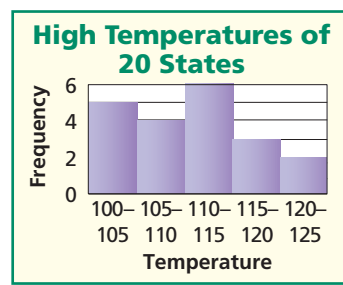- The distribution of the data can be determined from a histogram.

Example   Create a histogram to represent the following high temperatures in twenty states.

118   122   117   105   114   115   122   102   103   110
110   112   106   109   100   103   110   108   111   102

Since the temperatures range from 100 to 122, use measurement classes from 100 to 125 with 5 degree intervals. First create a frequency table and then draw the histogram.

Temperature Intervals	Tally	Frequency
$100 \leq d < 105$	Ɱ	5
$105 \leq d < 110$	IIII	4
$110 \leq d < 115$	Ɱ I	6
$115 \leq d < 120$	III	3
$120 \leq d < 125$	II	2

High Temperatures of 20 States

Exercises   Create a histogram to represent each data set.   *See Example 3 on page 724.*

23. the number of cellular minutes used last month by employees of a company

122   150   110   290   145   330   300   210   95   101   106   289   219
105   302   29   288   154   235   168   55   84   92   175   180

24. the number of cups of coffee consumed per customer at a snack shop between 6 A.M. and 8 A.M.

0 2 0 2 1 3 2 1 2 3 0 2 2 1 0 2 1 3 0 1 2 2
3 2 1 0 1 2 1 0 2 2 2 1 1 2 1 2 0 3 1 0 0 1

13-4 Measures of Variation

See pages 731–736.

Concept Summary

- The range of the data set is the difference between the greatest and the least values of the set and describes the spread of the data.
- The interquartile range is the difference between the upper and lower quartiles of a set of data. It is the range of the middle half of the data.
- Outliers are values that are much less than or much greater than the rest of the data.

Example   Find the range, median, lower quartile, upper quartile, and interquartile range of the set of data below. Identify any outliers.

25, 20, 30, 24, 22, 26, 28, 29, 19

Order the set of data from least to greatest.

19   20   22   24   25   26   28   29   30
      ↑   ↑   ↑
      Q_1 Q_2 Q_3

The range is $30 - 19$ or 11.   The median is the middle number, 25.

The lower quartile is $\dfrac{20 + 22}{2}$ or 21.   The upper quartile is $\dfrac{28 + 29}{2}$ or 28.5.

The interquartile range is $28.5 - 21$ or 7.5.

The outliers would be less than $21 - 1.5(7.5)$ or 9.75 and greater than $28.5 + 1.5(7.5)$ or 39.25. There are no outliers.

CONTENTS

Chapter

13 For More ...

• Extra Practice, see pages 849–850.
• Mixed Problem Solving, see page 865.

Exercises Find the range, median, lower quartile, upper quartile, and interquartile range of each set of data. Identify any outliers.
See Examples 1–3 on pages 731–733.

25. 30, 90, 40, 70, 50, 100, 80, 60

26. 3, 3.2, 45, 7, 2, 1, 3.4, 4, 5.3, 5, 78, 8, 21, 5

27. 85, 77, 58, 69, 62, 73, 55, 82, 67, 77, 59, 92, 75, 69, 76

28. 111.5, 70.7, 59.8, 68.6, 63.8, 254.8, 64.3, 82.3, 91.7, 88.9, 110.5, 77.1

13-5 *Box-and-Whisker Plots*

See pages 737–742.

Concept Summary

• The vertical rule in the box of a box-and-whisker plot represents the median.
• The box of a box-and-whisker plot represents the interquartile range.
• The bullets at each end of a box-and-whisker plot are the extremes.
• Parallel box-and-whisker plots can be used to compare data.

Example
The following high temperatures (°F) were recorded during a two-week cold spell in St. Louis. Draw a box-and-whisker plot of the temperatures.
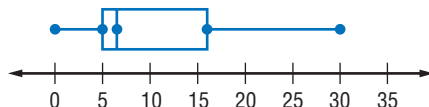
20	2	12	5	4	16	17
7	6	16	5	0	5	30

Order the data from least to greatest.

0 2 4 5 5 5 6 7 12 16 16 17 20 30

Q_1 $Q_2 = \frac{6+7}{2}$ or 6.5 Q_3

The interquartile range is $16 - 5$ or 11. Check to see if there are any outliers.

$5 - 1.5(11) = -11.5$ $16 + 1.5(11) = 32.5$

There are no outliers.

Exercises Draw a box-and-whisker plot for each set of data.
See Example 1 on page 738.

29. The number of Calories in a serving of French fries at 13 restaurants are 250, 240, 220, 348, 199, 200, 125, 230, 274, 239, 212, 240, and 327.

30. Mrs. Lowery's class has the following scores on their math tests.
60, 70, 70, 75, 80, 85, 85, 90, 95, 100

31. The average daily temperatures on a beach in Florida for each month of one year are 52.4, 55.2, 61.1, 67.0, 73.4, 79.1, 81.6, 81.2, 78.1, 69.8, 61.9, and 55.1.

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Vocabulary and Concepts

In a matrix, identify each item described.

1. a vertical set of numbers
2. an entry in a matrix
3. a horizontal set of numbers
4. a constant multiplied by each element in the matrix
5. number of rows and columns

a. element
b. column
c. row
d. dimensions
e. scalar

Skills and Applications

Identify the sample, suggest a population from which it was selected, and state whether it is *unbiased* (random) or *biased*. If unbiased, classify the sample as *simple*, *stratified*, or *systematic*. If biased, classify as *convenience* or *voluntary response*.

6. **DOGS**   A veterinarian needs a sample of dogs in his kennel to be tested for fleas. He selects the first 5 dogs who run from the pen.

7. **LIBRARIES**   A librarian wants to sample book titles checked out on Wednesday. He randomly chooses a book for each hour that the library is open.

If $W = \begin{bmatrix} 2 & 3 & 1 \\ -1 & 0 & -1 \\ 2 & -2 & 0 \end{bmatrix}$, $X = \begin{bmatrix} 4 & 2 & -1 \\ -2 & -2 & 0 \\ 0 & 1 & 2 \end{bmatrix}$, $Y = \begin{bmatrix} 3 & -2 & 1 \\ -1 & -2 & 4 \end{bmatrix}$, and $Z = \begin{bmatrix} 3 & 1 & 6 \\ 4 & -1 & -1 \end{bmatrix}$, find

each sum, difference, or product. If the sum or difference does not exist, write *impossible*.

8. $W + X$ 9. $Y - Z$ 10. $3X$ 11. $-2Z$ 12. $2W - Z$ 13. $Y - 2Z$

Create a histogram to represent each data set.

14. 68   71   74   90   81   72   71   69   65   92   75   69   71   73   73
    68   74   80   83   70   80   74   74   70   71

15. 10   40   50   52   22   50   60   90   41   51   90   40   75   63   53

Find the range, median, lower quartile, upper quartile, and interquartile range for each set of data. Identify any outliers.
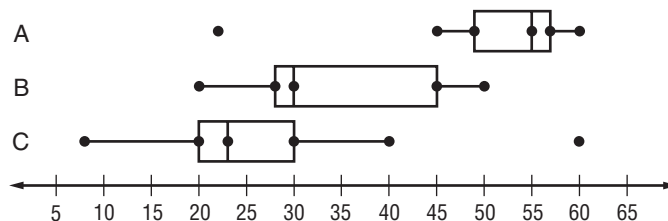
16. 1055, 1075, 1095, 1125, 1005, 975, 1125, 1100, 1145, 1025, 1075

17. 0.4, 0.2, 0.5, 0.9, 0.3, 0.4, 0.5, 1.9, 0.5, 0.7, 0.8, 0.6, 0.2, 0.1, 0.4

Draw a box-and-whisker plot for each set of data.

18. 1, 3, 2, 2, 1, 9, 4, 6, 1, 10, 1, 4, 5, 10, 1, 3, 6

19. 14, 18, 9, 9, 12, 22, 16, 12, 14, 16, 15, 13, 9, 10, 11, 12

NC Practice

20. **STANDARDIZED TEST PRACTICE**   Which box-and-whisker plot has the greatest interquartile range?

Ⓐ A   Ⓑ B   Ⓒ C   Ⓓ They all have the same interquartile range.

NC Practice

Part 1 Multiple Choice

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

1. Which equation represents a line perpendicular to the graph of $y = 4x - 6$? (Lesson 5-6)

   Ⓐ $y = \frac{1}{4}x + \frac{1}{6}$ Ⓑ $y = -\frac{1}{4}x + 2$

   Ⓒ $y = -4x + 6$ Ⓓ $y = 4x + 6$

2. A certain number is proportional to another number in the ratio 3:5. If 8 is subtracted from the sum of the numbers, the result is 32. What is the greater number? (Lesson 7-2)

   Ⓐ 15   Ⓑ 25

   Ⓒ 35   Ⓓ 40

3. The expression $(x - 8)^2$ is equivalent to (Lesson 8-8)

   Ⓐ $x^2 - 64$.   Ⓑ $x^2 - 16x + 64$.

   Ⓒ $x^2 + 16x + 64$.   Ⓓ $x^2 + 64$.

4. What is the least y value of the graph of $y = x^2 - 4$? (Lesson 10-1)

   Ⓐ 2   Ⓑ 0

   Ⓒ −2   Ⓓ −4

5. The expression $3\sqrt{72} - 3\sqrt{2}$ is equivalent to (Lesson 11-2)

   Ⓐ $3\sqrt{70}$.   Ⓑ $3\sqrt{2}$.

   Ⓒ $15\sqrt{2}$.   Ⓓ $5\sqrt{2}$.

6. A 12-meter flagpole casts a 9-meter shadow. At the same time, the building next to it casts a 27-meter shadow. How tall is the building? (Lesson 11-6)

   Ⓐ 20.25 m   Ⓑ 36 m

   Ⓒ 40 m   Ⓓ 84 m

7. Students are conducting a poll at Cedar Grove High School to determine whether to change the school colors. Which would be the best place to find an unbiased sample of students who represent the entire student body? (Lesson 13-1)

   Ⓐ a football practice

   Ⓑ a freshmen class party

   Ⓒ a Spanish class

   Ⓓ the cafeteria

8. A Mars year is longer than an Earth year because Mars takes longer to orbit the Sun. The table shows a person's age in both Earth years and Mars years. The data represent which kind of function? (Lesson 13-3)

Earth	10	20	30	40	50
Mars	5.3	10.6	15.9	21.2	26.5

   Ⓐ linear function

   Ⓑ quadratic function

   Ⓒ exponential function

   Ⓓ rational function

Use the box-and-whisker plot for Questions 9 and 10.

Miles per Gallon of Four Different Cars
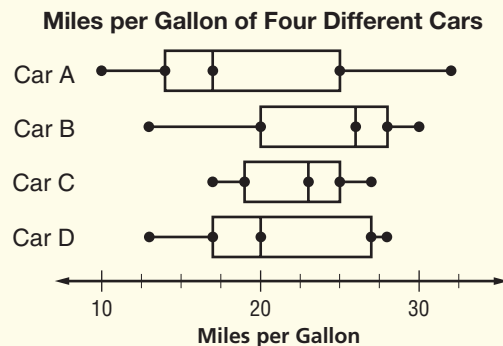
9. Which car shows the least variation in miles per gallon? (Lesson 13-5)

   Ⓐ A   Ⓑ B   Ⓒ C   Ⓓ D

10. Which car model has the highest median miles per gallon? (Lesson 13-5)

   Ⓐ A   Ⓑ B   Ⓒ C   Ⓓ D

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Part 2 Short Response/Grid In

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

11. Factor $x^3 + 8x^2 + 16x$. **(Lesson 9-3)**

12. Solve $6x^2 + x - 2 = 0$ by factoring.
(Lesson 9-4)

13. Simplify $\sqrt{4\sqrt{9}}$. **(Lesson 11-1)**

14. Maren can do a job in 4 hours. Juliana can do the same job in 6 hours. Suppose Juliana works on the job for 2 hours and then is joined by Maren. Find the number of hours it will take both working together to finish the job. **(Lesson 11-4)**

15. The map below shows train tracks cutting across a grid of city streets. Newton Street is 1.5 miles apart from Olive Street, Olive Street is 1.5 miles apart from Pine Street, and the three streets are parallel to each other. If the distance between points A and B is 5 miles, then what is the distance in miles between points B and C? **(Lesson 12-4)**
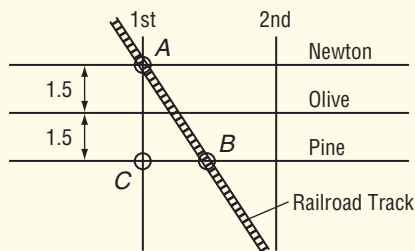
Part 3 Quantitative Comparison

Compare the quantity in Column A and the quantity in Column B. Then determine whether:

Ⓐ **the quantity in Column A is greater,**

Ⓑ **the quantity in Column B is greater,**

Ⓒ **the two quantities are equal, or**

Ⓓ **the relationship cannot be determined from the information given.**

Column A	Column B
16. the sum of the next three terms of the arithmetic sequence $-250, 83, 416, \ldots$	the 67th term of the arithmetic sequence $-49, 2, 53, \ldots$

(Lesson 4-7)

Column A	Column B
17. the root of $y = -0.25x^2 + x - 1$	the sum of the roots of $b = 3a^2 - 5a + 2$

(Lesson 10-4)

Column A	Column B
18. the value of x if $\dfrac{3x + 1}{14} = \dfrac{x}{4}$	the value of y if $\dfrac{45}{4y + 1} = \dfrac{10}{y}$

(Lesson 12-9)

Part 4 Open Ended

Record your answers on a sheet of paper. Show your work.

19. Construct a histogram for the following data. Use intervals of 40–50, 50–60, 60–70, 70–80, 80–90, and 90–100. **(Lesson 13-3)**
45, 62, 78, 84, 63, 73, 68, 91, 65, 80,
71, 87, 85, 77, 78, 80, 83, 87, 90, 91

20. In Exercise 19, what percent of the data lies within the tallest bar? **(Lesson 13-3)**

21. Draw a box-and-whisker plot of the following test scores. **(Lesson 13-5)**
24, 38, 47, 22, 40, 36, 25, 48, 30, 32, 45, 41, 34, 39, 40, 47, 40, 38, 42, 49, 45

www.algebra1.com/standardized_test/nc

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